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GRAFTED GENETIC ALGORITHM AND THE TRAVELING VISITOR
PROBLEM

PhD Thesis

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Dedicated to my family!

Abstract

Grafted Genetic Algorithm and the Traveling Visitor Problem

In this PhD Thesis two related topics from theoretical computer science are considered. The first one considers Grafted Genetic Algorithms. The thesis analyzes separate, combined and partial performance of local searcher and genetic algorithm. On the Traveling Salesman Problem we examine the impact of hybridization a 2-opt heuristic based local searcher into the genetic algorithm. Genetic algorithm provides a diversification, while 2-opt improves intensification. Results on examples from TSPLIB show that this method combines good qualities from both methods applied and significantly outperforms each individual method. In tests we applied hybridization at various percentages of genetic algorithm iterations. We also studied at which iterations of the genetic algorithm to apply the hybridization. We applied it at random iterations, at the initial iterations, and the ending ones where the later proved to be the best.

The second topic considers the Traveling Visitor Problem. We consider the problem when a visitor wants to visit all interesting sites in a city exactly once and to come back to the hotel. Since, the visitors use only walking trails and pedestrian zones, the goal is to minimize the traveling visitor's walk. A new problem the Traveling Visitor Problem is then similar to the Traveling Salesman Problem with a difference that the traveling visitors, during its visit of sites, can not fly over buildings in the city, instead visitors have to go around these obstacles. That means that all Euclidean distances, like those in Euclidean TSP, are impossible in this case. The tested benchmarks are combined from three real instances made using tourist maps of cities of Koper, Belgrade and Venice and two instances of modified cases from TSPLIB. We compared two methods for solving the Traveling Visitor Problem. In all tested cases the Koper Algorithm significantly outperforms the Naïve Algorithm for solving the TVP.

ACM Computing Classification System: F.2.1, G.1.2, G.1.6, G.2.2, G.2.3.

Key words: Traveling Salesman Problem, Genetic Algorithms, Memetic Algorithms, Grafted Genetic Algorithms, Traveling Visitor Problem, Koper Algorithm, Naïve Algorithm.

Izvleček

Cepljeni genetski algoritmi in problem potujočega obiskovalca

V doktorski disertaciji sta obravnavani dve povezani temi s področja teoretičnega računalništva.

Tretje poglavje obravnava cepljene genetske algoritme. Disertacija analizira skupno in delno izvedbo lokalnega iskalca in genetskega algoritma. Na problemu trgovskega potnika (angl. Traveling Salesman Problem), smo preučevali vpliv hibridizacije 2-opt hevrističnega lokalnega iskalca v genetski algoritem. Genetski algoritem zagotavlja raznolikost (angl. diversification), medtem ko 2-opt izboljša krepitev (angl. intensification). Rezultati primerov iz TSPLIB kažejo na to, da metoda združuje dobre lastnosti obeh metod in znatno prekaša vsako posamezno metodo. V testih smo uporabili hibridizacijo pri različnih odstotkih genetskih iteracij algoritma. Prav tako smo preizkušali, katere iteracije genetskega algoritma je potrebno hibridizirati. Hibridizacijo smo izvedli pri naključnih, začetnih in končnih iteracijah, za katere se je kasneje izkazalo, da so najboljše.

Četrto poglavje obravnava problem potujočega obiskovalca (angl. Traveling Visitor Problem). Problem nastane, ko si obiskovalec želi ogledati vse zanimive lokacije v mestu natanko enkrat in se po ogledu vrniti v hotel. Cilj je zmanjšati sprehod potujočega obiskovalca.

Problem potujočega obiskovalca je izpeljan iz problema trgovskega potnika, pri čemer velja pravilo, da obiskovalec izbira samo med potmi, ki jih je možno prehoditi. To pomeni, da so Evklidske razdalje (angl. Euclidean distance), kot jih poznamo v Evklidskem TSP, v našem primeru nemogoče. Obiskovalci uporabljajo sprehajalne poti in območja za pešce, ki so različno dolge. Te omejitve določajo teže povezav, ki povezujejo vozlišča v grafu. Za testiranje smo uporabili primere problema potujočega obiskovalca, ki smo jih izdelali iz uradnih turističnih zemljevidov za mesta Koper, Beograd in Benetke ter dveh spremenjenih primerov iz TSPLIB. Za reševanje problema potujočega obiskovalca smo primerjali dve metodi. V vseh testiranih primerih problema potujočega obiskovalca, algoritem koper občutno prekaša naivni algoritem.

ACM Computing Classification System: F.2.1, G.1.2, G.1.6, G.2.2, G.2.3.

Ključne besede: problem trgovskega obiskovalca, genetski algoritmi, memetični algoritmi, cepljeni genetski algoritmi, problem potujočega obiskovalca, algoritem koper, naivni algoritem.

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Chapter 1

Introduction

I arrived to Koper, from Belgrade, on postgraduate studies of computer science, in 2007. Immediately, I had an urge to acquaint myself with the city in which I was to live for the next four years. I was drawn by the city center with its numerous sites - exactly 55 of them were listed in the official tourist map of Koper. Since doctoral studies were exhausting, I did not have much spare time to roam the streets of Koper. So I began to wonder whether I could devise a way of optimizing the tour through the city, by visiting all sites in as few steps as possible. We named the problem the *Traveling Visitor Problem* (TVP).

1.1 Traveling Visitor Problem

The Traveling Visitor Problem is a version of the *Traveling Salesman Problem*, (TSP) [5, 55, 60, 63, 66, 69, 75, 92, 99], with a difference that the traveling visitors, during their visit of sites, can not fly over the buildings in the city, instead visitors must go around these obstacles. This difference is demonstrated in Figure 1.1. This means that direct edge connections, as we know them in the TSP, are in this case impossible (direct edge from i to j in Figure 1.1). Visitors use the walking paths and pedestrian zones of variable length. These limits determine the weight of edges connecting the vertices in the graph.

Formally, the Traveling Visitor Problem is stated as: given a connected, weighted

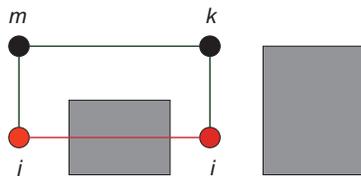


Figure 1.1: TSP and TVP

graph $G = (V, E, c)$, with a set of vertices $V = S \cup X$ and $S \cap X = \emptyset$, S belongs to interesting sites in a city, X belongs to crossroads in a city, a set of edges E , and a cost of traveling c . The goal is to find the shortest closed walk through all vertices of S , according to c in graph G , although we may walk through vertices from X .

Chapter 2 introduces the basic concepts of the Traveling Salesman Problem and related problems. Furthermore, the algorithms for solving it are described. This chapter, provides the knowledge necessary for further chapters of the dissertation to become understandable to the broad range of readers. Furthermore, it presents an interpretation of the various textbooks which represents the state of the art literature on related areas of computer science.

1.2 Traveling Salesman Problem

In the early 30's of the 20th century, the Austrian mathematician Karl Menger challenged the research community of that time to consider, from the mathematical point of view, the following problem: A traveling salesman has to visit exactly once each one of a list of n cities and then return to the home city. He knows the cost of traveling from any city i to any other city j . Thus, the question is, which is the tour of least possible cost the salesman can take [100].

The instance of the TSP is formally defined on the complete graph G , with the set of vertices $V = \{v_1, v_2, \dots, v_n\}$, for some integer n and by a cost function assigning a cost $c_{i,j}$ to the edge (v_i, v_j) for any i and j in G .

TSP can be viewed also as a permutation problem. Let P_n be a set of all permutations of the set $\{1, 2, \dots, n\}$. Then the traveling salesman goal is to find a permutation $\pi = (\pi(1), \pi(2), \dots, \pi(n))$ in P_n , that minimizes the quantity

$$\sum_{i=1}^{n-1} d(c_{\pi(i)}, c_{\pi(i+1)}) + d(c_{\pi(n)}, c_{\pi(1)}). \quad (1.1)$$

This quantity is referred to as the tour length since it is the length of the tour a salesman would make when visiting the cities in the order specified by the permutation π , returning at the end to the initial city.

TSP is one of the most important representatives of a large class of problems known as combinatorial optimization problems [67]. Since the TSP belongs to a class of *NP-hard* problems [76], an efficient algorithm for TSP probably does not exist. More accurately, such an algorithm exists if and only if the two computational classes P and NP coincide. From a practical point of view, it means that in general it is quite impossible to find an exact solution for any TSP instance with n nodes, that has a behaviour considerably better than the algorithm which computes all of the $(n - 1)!$ possible distinct tours, and then returns the least costly one.

If we are looking for applications, a different approach can be used. Given a TSP instance with n nodes, any tour passing once through all cities is a feasible solution. Algorithms that construct in polynomial time with respect to n feasible solutions are called heuristics [12, 121]. In general, these algorithms produce solutions but without any insights to how far is their cost from the optimal.

There exist two versions of TSP: the Symmetric and the Asymmetric TSP. In the symmetric form, known as the STSP [73, 74, 93, 97], the cost distance between nodes i and j is equal to the cost distance between nodes j and i ($c_{ij} = c_{ji}$). In the case of asymmetric TSP (ATSP) [14, 16, 18, 24], there is no such symmetry. In addition, there are many different variations of TSP which are described and explored in the literature and also a variations derived from everyday life. From the dissertation point of view, the most important applications of the TSP studied in the literature, are in more details presented in Chapter 2.

First steps in solving the TSP consist of exact methods and heuristics. Exact methods like cutting planes [115] and branch and bound [27, 115], can optimally solve relatively small problems (depending on the size of n), while methods such as different variants of Lin-Kernighan algorithm [6, 46, 70, 79], and Concorde algorithm [3–5] are good for larger problems. Furthermore, some algorithms based on greedy principles such as nearest neighbour [63], and spanning tree [69] can be also used for solving the TSP. The above-mentioned methods for solving TSP result in exponential computational complexities. For that reason a new methods are required to overcome this shortcoming. These new methods include different kinds of heuristic techniques, nature based optimization algorithms, etc. Various creatures and natural systems, developed in nature, are interesting and valuable sources of inspiration for exploring and creating new methods for solving the TSP and variations of TSP. Let us count the most important of them. Evolutionary Algorithms [101, 106, 127, 136], Genetic Algorithms [43, 51, 52, 103, 109, 117, 126, 130, 134, 135, 139, 140], Memetic Algorithms [62, 87–89, 104, 113], Simulated Annealing [84], Ant Systems [39] and finally Grafted Genetic Algorithms [35], [38], [36], [37]. The latter is a type of hybrid genetic algorithms [43, 68, 94, 139] and they will be described and demonstrated in the thesis.

The first topic of the thesis, presented in Chapter 3, deals with the heuristic method called the Grafted Genetic Algorithm, through which we solve the Traveling Salesman Problem. In this chapter the aim is to show the quality of the solution and the running time of the grafted genetic algorithm when it is applied to the instances of the problems of symmetric TSP which are available on the Internet.

The second topic of the thesis, presented in Chapter 4, elaborates the Traveling Visitor Problem. This chapter describes a problem and examples of real cities and finally solves the instances of the problem by using new methods.

1.3 Research Objectives

Research objective of the thesis is to prove the following hypotheses:

- Hypothesis 1: The method for solving of TSP that is made of two independent methods, genetic algorithm and 2-opt heuristic, outperforms each of the combined methods in terms of the quality of solution.
- Hypothesis 2: The quality of solution of a proposed method to the problem of a traveling visitor problem, outperforms algorithms for solving general TSP problem, when they are used for solving traveling visitor problem.

Proofs of hypothesis 1 and the hypothesis 2 are in the Chapter 3 and Chapter 4, respectively.

Contributions to the science consist of the following results:

- Construction of the grafted genetic algorithm for solving the traveling salesman problem.
- Verification that the traveling salesman problem can be successfully solved using the grafted genetic algorithm.
- Construction of the method for solving the traveling visitor problem.
- Construction of the solutions of the traveling visitor problem for cities of Koper, Belgrade and Venice.
- Verification that all instances of the traveling visitor problem, which are solved using the proposed method, represent a very satisfactory solution.

The results of the thesis represents the contribution to bridging the gap between theoretical computer science and its application in practice. Also to better understanding and modeling of real problems in the economy, represented as the NP-hard problems from graph theory as well as a contribution to the optimization methods for solving these problems.

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- M. Djordjevic, and A. Brodник, Quantitative Analysis of Separate and Combined Performance of Local Searcher and Genetic Algorithm, *Book of Abstract of International Conference on Operations Research, OR 2011, Zurich, Switzerland* (2011), 130.
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Chapter 2

Background

2.1 Traveling Salesman Problem

In this chapter we introduce the basic concepts of optimization and related combinatorial problems. Among those problems, probably best known is the Traveling Salesman Problem [5, 55, 60, 63, 64, 66, 69, 75, 92, 99], which we describe in this section in more detail. We illustrate the computational properties of the Traveling Salesman Problem which is closely related to many optimization problems arising in real world applications. We refine some graph theoretical approaches, which can be applied when attempting to tackle traveling salesman like problems. Traveling Salesman Problem can be stated as follows. Including his home town, a salesman wants to visit n cities and then return home. The objective is to find a tour for visiting each city exactly once while minimising the total distance traveled. The problem of finding a minimal tour is equivalent to finding an optimal ordering of the set of cities. The TSP is consequently often formulated as a permutation problem. Given a set of n cities $\{c_1, c_2, \dots, c_n\}$ and for each pair of cities (c_i, c_j) a distance $d(c_i, c_j)$. The goal is then to find a permutation π of the cities which minimises the length of the tour given by:

$$\sum_{i=1}^{n-1} d(c_{\pi(i)}, c_{\pi(i+1)}) + d(c_{\pi(n)}, c_{\pi(1)}). \quad (2.1)$$

As mentioned before, the TSP has attracted the focus of researchers for decades. According to a survey on the history of combinatorial optimization by [124], the TSP was formulated as early as 1832 in a German manual for the successful traveling salesman, before it was presented as a research problem by Menger in the 1930's. Since then, the TSP's simplicity on the one hand and the difficulty of finding optimal solutions on the other, has affirmed it as a test bed for new heuristics and exact algorithms. But, the problem has not only theoretical foundations and there are many real life applications of the Traveling Salesman Problem and its variants [78, 114], some of which we will describe later in this chapter. Let us now introduce the Traveling Salesman Problem in mathematical terms as an integer programming

problem. Take up variables $x_{i,j}$ which are either equivalent to one if the city j is straightly visited after city i or zero otherwise. The Traveling Salesman Problem may be established as an integer program [83] as follows:

$$\begin{aligned}
 & \text{Minimise } \sum_{i=1}^n \sum_{j=1}^n d(c_i, c_j) x_{i,j}, \\
 & \text{Subject to } \sum_{i=1}^n x_{i,j} = 1, \forall j = 1, 2, \dots, n, \\
 & \sum_{j=1}^n x_{i,j} = 1, \forall i = 1, 2, \dots, n, \\
 & \sum_{i \in I} \sum_{j \notin I} x_{i,j} \geq 1, I \subset \{1, 2, \dots, n\} \text{ with } 1 < |I| < n, \\
 & \text{and } x_{i,j} \in \{0, 1\}, \forall i, j = 1, 2, \dots, n,
 \end{aligned} \tag{2.2}$$

Here, our goal is to give the sum of all combination variables $x_{i,j}$ weighted by the lengths between the cities. This function is subject to a number of constraints. The constraints stated first and second inflict each city to be visited exactly once as a part of the tour. The constraints denoted third are the so called sub-tour avoidance constraints which are needed to guarantee that the solution represents a connected tour. The presentation of the Traveling Salesman Problem as an integer program shown above is not unique; there are various representations, see for example [112]. In the next subsections we will present some important notions for optimization problems and the Traveling Salesman Problem, together with a review of problems similar to the TSP.

2.1.1 Graph representation of the TSP

Traveling Salesman Problem is ordinarily represented and considered as a graph theoretical problem [78,83]. This representation is suitable because many algorithms for the TSP are established on graph theoretical concepts. Here, we present some graph theoretical definitions needed to represent the Traveling Salesman Problem in graph theoretical terms.

An undirected graph $G = (V, E)$ consists of a finite set of vertices (or nodes) V and a finite set of edges E . Each edge $e \in E$ correspond to a set $e = \{u, v\}$ of two vertices $u, v \in V$. An edge $e = \{u, v\}$ is *incident* to vertices u and v and the number of edges incident to a vertex is the *degree* of the vertex. In the situation where there exists an edge $e = \{u, v\}$ between each pair $u, v \in V$ we speak of a *complete graph*.

Now, some concepts of the Traveling Salesman Problem will be presented. If $P = (\{u_1, u_2\}, \{u_2, u_3\}, \dots, \{u_{k-1}, u_k\})$, where $\{u_i, u_{i+1}\} \in E$, then P is called a *walk*. If further $u_i \neq u_j$ for all $i \neq j$ it is called a *path*. A closed path, where $P = (\{u_1, u_2\}, \{u_2, u_3\}, \dots, \{u_k, u_1\})$, i.e. a path as well as the edge returning to the vertex it started from, is denoted to as a *cycle*. Graphs are generally categorised according to their properties. Two important classes, for the Traveling Salesman

Problem are the *Eulerian* and the *Hamiltonian graphs*. A graph is called *Eulerian* if it contains an *Eulerian Tour*, which is a closed walk traversing every edge of the E in graph G . A graph is called *Hamiltonian* if there exists a cycle which visit every vertex from V in graph G . Those cycles are called *Hamiltonian Cycles*. It is worthy to measure the qualities of a walk, path or tour in comparison to others. Furthermore, *weights* are associated with each edge of the graph. For such a graph we say a *weighted graph*. Because a walk, path and tour can be identified by the set of edges traversed, P , its quality can be valuated by a weight or cost function $c : P \rightarrow \mathbb{Q}$, which will map the edge set P into the set of the rational numbers \mathbb{Q} . Let $c_{i,j}$ refer to the weight of edge $\{i, j\}$, then the weight of an edge set P is then specified as the sum over all the edges of P :

$$\sum_{\{i,j\} \in P} c_{i,j}. \quad (2.3)$$

For a tour or a path the sum of weights of its edges is commonly specified to as its *length*. Let us now determine the symmetric TSP (STSP) [73, 74, 93, 97]. In the STSP the distance between nodes i and j is equal to the distance between nodes j and i . An instance of a TSP can be seen as a complete graph $G = (V, E)$ where the set of vertices V is given by the cities and edges in the graph with corresponding edge weights $c_{i,j}$ are given by the distances between cities. The Traveling Salesman Problem is then equal to the problem of finding a Hamiltonian Tour of minimal length in the graph G . For many applications it is useful to correlate a direction of the edges of a graph. Because the weights between vertices are not necessary symmetric, i.e. $c_{i,j} \neq c_{j,i}$, for some edges of G . This kind of graph is called *directed graph* or *digraph* and its edges are ordered 2-tuples of vertices, referred to as *arcs*. The asymmetric Traveling Salesman Problem (ATSP) [14, 16, 18, 24] is then similar to the symmetric TSP above, i.e. it is the problem of finding a Hamiltonian Tour of minimal length in a complete digraph which also take the directions of the edges into account. The *Euclidean TSP*, or planar TSP, is the TSP with the distance being the ordinary *Euclidean distance*. The Euclidean TSP [57, 113] is then a particular case of metric TSPs, since distances in a plane obey the triangle inequality.

2.1.2 Computational complexity of the TSP

As was mentioned at the start of this section the Traveling Salesman Problem is a very hard combinatorial optimization problem. In order to evaluate algorithms according to their computational requirements the theory of *Computational Complexity* has been developed. In plain terms, the computational complexity deals with the number of computational steps an algorithm needs to solve an instance of a problem of size n . An algorithm can be described as a "step-by-step procedure" [76]. The number of computational steps realized present a measure of the execution time needed to solve an instance of a problem. The number of steps an algorithm requires, is often not only dependent on the size of the problem instance. It may also differ between instances of the same size. Furthermore, it is not always straightforward to estimate the number of steps an algorithm needs for a given instance. To be able to analyse the complexity of an algorithm, a *worst-case analysis* is introduced. The

computational complexity is defined as the *maximum number of steps* an algorithm may require for any instance of a given size.

To recap, the performance of an algorithm is measured as the maximum number of steps it requires for any problem instance of size n denoted as a function of n . Widely adopted concept is that an algorithm is effective if its worst-case complexity is bounded by a polynomial function of instance size n while algorithms of exponential time complexity are commonly considered ineffective or computationally expensive [76]. Even though an exponential function may initially give smaller values than a given polynomial, there will be always a constant N such that for all $n \geq N$ the polynomial function get lesser values than the exponential.

The Classes P and NP

The outline summarized in previous section can be used to categorize an algorithm as efficient or not dependent upon whether it has polynomial time complexity. Similarly a problem is considered easy or hard depending whether there exists a polynomial time algorithm for solving it. Furthermore, it means that a problem is considered easy if there is an algorithm where in the worst case the number of steps of the problem of size n is bounded by a polynomial of n . The main purpose to investigate the computational complexity of decision problems is that there exist efficient methods for concerning the complexities of different decision problems. Decision Problems consist of an instance of a problem and a question to which the answer is 'yes' or 'no'. An example of a decision problem related to the TSP is the *Hamiltonian Cycle Problem* [76]:

Problem 2.1.1 *Instance:* Graph $G = (V, E)$ *Question:* Does there exist a cycle in G passing through each vertex in V exactly once?

As we described above, looking from the graph theoretical point of view, the TSP is equivalent to finding a Hamiltonian cycle of a minimum length in a complete graph G . As a consequence, for the TSP decision problem the main question is not whether a cycle exists in a complete graph G , but if there exists a cycle of length less than a given constant B . The TSP Decision Problem can be stated as follows [76]:

Problem 2.1.2 *Instance:* Given a complete weighted graph $G = (V, E)$ with non-negative edge weights w_i , for $i \in E$ and a constant $B \geq 0$ *Question:* Does there exist a Hamiltonian cycle in G (or a Tour of a Traveling Salesman) with edge sequence S where $\sum_{i \in S} w_i \leq B$?

It is easy to see that if there exists a polynomial time algorithm for the TSP, there also exists a polynomial time algorithm for the TSP Decision Problem. On the other hand, it is not so evident and even not always true that the converse holds as well. A polynomial time algorithm for generating the optimal TSP tours by calling a subroutine that solves the TSP Decision Problem is introduced by Johnson and Papadimitriou in [76]. Supposing that the TSP decision problem can be solved in polynomial time, this algorithm, provides a polynomial time algorithm for the Traveling Salesman Problem. This represents a type of polynomial time reduction [33]. Additionally the presence of this algorithm proves the following theorem [76]:

Theorem 2.1.3 *There exists a polynomial time algorithm for the TSP if and only if there exists a polynomial time algorithm for the TSP Decision Problem.*

This theorem indicates that when we are interested in the computational complexity it is adequate to restrain focus to the TSP Decision Problem. That's why the question of whether the TSP is a "hard" to solve problem is equivalent to whether the TSP Decision Problem is a "hard" problem. Let us now introduce the two classes of problems in the theory of computational complexity. The class P contains all decision problems for which exists a polynomial time algorithm. Many combinatorial optimization problems are not in P . Nevertheless, it is not easily shown that certain problems are not in P even if no polynomial time algorithm is known. To specify the second class of algorithms we must first define the concept of *non-deterministic* algorithms.

Opposite to the deterministic algorithm, a non-deterministic algorithm can exhibit different behaviors on different runs. The class NP are composed of all decision problems, which can be solved in polynomial time by a non-deterministic algorithm. It is obvious that $P \subseteq NP$. In spite of that, the question of whether $P = NP$ denotes one of the major questions of the computer science community. A lot of research has been spent on this challenge over the last five to six decades and the fact that no polynomial-time algorithm for specific problems in NP has been found, makes the equivalence of both classes very unlikely. Because of this, it is widely assumed that $P \neq NP$, even though this conjecture remains unproven to present date.

The Class NP-Complete

Very important subset of problems in NP is the class of *NP-complete* problems. The most important property of NP-complete decision problems is that their computational status is directly connected to the relation between P and NP . The concept of NP-completeness was introduced in [26]. Regrettably, concerning the Traveling Salesman Problem, it has been shown that it belongs to the class of NP-complete problems. As Johnson and Papadimitriou comment in [76]:

This is our main negative result for the TSP; it is the strongest negative result one can hope to prove, short of establishing $P \neq NP$.

In contrast to the consideration that the TSP is not NP-complete as it is not a decision problem. Before in the chapter we have mentioned a polynomial time algorithm for the TSP which directly leads to a polynomial time algorithm for the TSP Decision Problem and also its NP-completeness implies that $P = NP$. Problems which have this property are referred to as *NP-hard* problems.

2.1.3 Applications of the TSP

The TSP variations and the applications of the TSP exceed the route planning problem of a traveling salesman and cross over some areas of science including computer science, mathematics, operations research, genetics, engineering, and electronics. In the next three subsections we summarize, from the dissertation point of view, the most important applications of TSP studied in the literature.

Machine Scheduling Problem

Maybe the most studied application field of the Traveling Salesman Problem is scheduling and machine sequencing. A scheduling can be described as follows. There are n assignments $\{1, 2, \dots, n\}$ to be processed consecutive on a machine. Let c_{ij} , be the setup cost necessary for processing assignment j instantly after assignment i . When all assignments are processed, the machine is reset to its original state at a cost of $c_{j,l}$, where j is the last assignment processed. Therefore the machine sequencing problem is of finding an order in which the assignments are to be processed so that the total setup cost is minimized. Obviously, finding a permutation π of $\{1, 2, \dots, n\}$ that minimizes Equation 2.4 solves the problem:

$$c_{\pi(n)\pi(1)} + \sum_{i=1}^{n-1} c_{\pi(i)\pi(i+1)} \quad (2.4)$$

Looking from the practical point of view the assignments can be often clustered together. In this case the setup time, if any, between assignment within a cluster is relatively small compared to setup time between jobs in two different clusters. For example this can be a typical scenario in assembly line. Assembling the similar products need minimal setup time. However, if a different product needs to be assembled, the setup time may grow larger because it may require the new parts, tools etc. So the machine sequencing problem with such a specialized matrix of costs reduces to the *Clustered TSP*. This problem will be, in more detail, presented latter in desertation.

Now look at another scheduling problem - a *no wait flow shop* problem. There are n assignments each demanding processing on m machines in the order $1, 2, 3, \dots, m$. No assignment is approved to take a waiting time between processing on two sequential machines. The goal is to find an optimum sequencing of assignments which will be processed so that the total completion time is minimal. Applications of this kind of sequencing problems can arise in a different situations, more details can be found at [44]. Furthermore, the no-wait flow shop problem is strongly NP-hard, but solvable in polynomial time, when $m = 2$. In this particular case it reduces to the well known *Gilmore-Gomory* problem [63].

Taking an instance of the no-wait flow shop problem, it is known than it can be constructed an equivalent instance of TSP, by using the reduction proposed in [44]. So reduction rests in creation of a complete graph G on $n + 1$ nodes, in which nodes represent an assignment j , $1 \leq j \leq n$ and node $n + 1$ represents the start and the end of processing. The cost $c_{i,j}$ of edge (i, j) in G represents the additional schedule length, if assignment j is the direct successor of assignment i in the schedule. So a minimum cost tour in G represent a schedule with minimum completion time. To finish the reduction, the values of $c_{i,j}$ must be identified. Let $p_{j,k}$ be the processing time of assignment j on processor k , and also $1 \leq j \leq n$, $1 \leq k \leq m$. Then $c_{i,j}$ can

be obtained using the equations [44]:

$$\begin{aligned}
 c_{n+1,i} &= \sum_{r=1}^m p_{ir}, i = 1, 2, \dots, n, \\
 c_{ij} &= \max_{1 \leq k \leq m} \left\{ \sum_{r=1}^k p_{ir} + \sum_{r=k}^m p_{jr} \right\} - \sum_{r=1}^m p_{ir}, 1 \leq i, j \leq n, i \neq j \\
 c_{i,n+1} &= c_{ii} = 0, i = 1, 2, \dots, n.
 \end{aligned} \tag{2.5}$$

These applications that we considered so far reduce a given problem to the Traveling Salesman Problem.

For details of other research works on printed circuit board assembly that are relevant to the TSP, but goes beyond the scope of the dissertation, refer to [8, 63].

General Routing Problem

Given a connected, undirected graph $G = (V, E)$, that consists of a finite set of vertices V and a finite set of edges E , a cost c_e for each edge $e \in E$, a finite set $V_R \subseteq V$ of required vertices and a finite set $E_R \subseteq E$ of required edges. The General Routing Problem (GRP) is the problem of finding a minimum cost route passing through each $v \in V_R$ and each $e \in E_R$ at least once [111].

The GRP contains several other important routing problems as special cases:

- The *Rural Postman Problem* (RPP) is obtained when $V_R = \emptyset$. For more details on this problem, we refer to [111].
- The *Chinese Postman Problem* (CPP) is obtained when $V_R = \emptyset$ and $E_R = E$. For more details on this particular problem, we refer to [25].
- The *Steiner Graphical Traveling Salesman Problem* (SGTSP) is obtained when $E_R = \emptyset$. The SGTSP has been discussed in [29]. Moreover, in [47] this problem was also called the *Road Traveling Salesman Problem* (RTSP).
- The *Graphical Traveling Salesman Problem* (GTSP) is obtained when $E_R = \emptyset$ and $V_R = V$. For more details we refer to [29].

Frequency Assignment Problem

If we look a communication network together with a set of transmitters, *the Frequency Assignment Problem* is to specify a frequency to each transmitter from a given set of usable frequencies. The frequencies must satisfying some interference constraints. These constraints can be denoted by a graph $G = (V, E)$ in which each node denotes a transmitter. A non-negative integer weight c_{ij} is appointed for each arc (i, j) denoting the tolerance. Take $F = \{0, 1, 2, \dots, R\}$ to be a collection of usable frequencies. A frequency assignment is then to assign the number $f(i) \in F$ to node $i \in V$ such that $|f(i) - f(j)| > c_{ij}$ for all $(i, j) \in E$. In the case that such an assignment exists then it is called a feasible assignment. Note that if R is large

enough a feasible assignment is possible. Let us consider the following, let G^* be the complete graph obtained from G by adding edges of zero weight. Let $c'_{i,j}$ be the weight of edge (i, j) in G^* such that $c'_{i,j} = c_{i,j} + 1$. Finally, let $C'(H^*)$ be the sum of the weights of edges in a minimal cost Hamiltonian path in G^* . Then the TSP can be used to compute a lower bound for the frequency assignment problem, as it has been shown in [63].

2.1.4 Most Important Variations of the TSP Studied in the Literature

Our discussion on couple of next variations are confined to their definitions only and practical implementations for some variations. For more comprehensive overview on these particular problems, see the corresponding references cited. Note also that the references we cite are not necessarily a paper in which the problem was introduced. Below we summarize, from the dissertation point of view, the most important variations of TSP studied in the literature.

The time dependent TSP: For each arc (i, j) of graph G , n of different costs $c^t_{i,j}$, $t = 1, 2, \dots, n$ are given. The cost $c^t_{i,j}$ represent the cost of traveling from city i to city j in some time period t . The goal is to find a tour $(\pi(1), \pi(2), \dots, \pi(n), (1))$, where $\pi(1) = 1$ represents the home location, which is in the time period zero, in graph G such that $\sum_{i=1}^n c^i_{\pi(i)\pi(i+1)}$ is minimized. The index $n + 1$ is equivalent to 1. When all arcs (i, j) , are $c^1_{i,j} = c^2_{i,j} = \dots = c^n_{i,j}$ then this problem reduces to the TSP. For more details on this particular problem, we refer to [59].

Black and White TSP: The black and white TSP is a generalization of the Traveling Salesman Problem. In this problem, the set of nodes of G is partitioned into two sets, B and W . The elements of set B are addressed as black nodes and the elements of set W are addressed as white nodes. For tour H in G to be feasible the following two conditions must be satisfied. Firstly, the number of white nodes between any two sequential black nodes should not go beyond a positive integer I . Secondly, the distance between any two sequential black nodes should not go beyond a positive real number R . Then, the goal of the black and white TSP is to find a minimal cost feasible tour in graph G . It is important to note the practical implementation of this problem. Accordingly the applications of this problem involve design of ring networks in telecommunications [63]. Furthermore, a variation of this problem known as TSP with replenishment arcs, has been applied in the air-line industry. The TSP with replenishment arcs has been discussed in [98].

The delivery man problem: The delivery man problem (DMP) is also known as the *minimum latency problem* [11] and the *traveling repairman problem* [53]. Let H be a tour in G and v_1 be a starting node. For each vertex v_i of G , define the *latency* of v_i with respect to H expressed by $L_i(H)$, which represents the total distance in H from v_1 to v_i . The goal of the delivery man problem is then to find a tour H^* in graph G such that $\sum_{i=1}^n L_i(H^*)$ is minimized. Additionally it can be verified that this problem is a special case of the time dependent TSP described above. For more details on this problem, we refer to [63].

Clustered TSP: In this variation of TSP, the set of vertices of graph G is splitted into clusters V_1, V_2, \dots, V_k . The goal of clustered TSP tries to find a minimum cost

tour H in graph G with the constraint that vertices within the same cluster must be visited consecutively. Note also that by adding a large cost M to the cost of each inter-cluster edge this problem can be reduced to a Traveling Salesman Problem. More details about this problem can be found in [65].

Generalized TSP: As in the case of clustered TSP, let V_1, V_2, \dots, V_k be a partition of the set of vertices of graph G . In a Generalizes TSP (GTSP), the goal is to find a shortest cycle in graph G which goes through exactly one vertex from each cluster $V_i, 1 \leq i \leq k$. Note also that if $|V_i| = 1$ for all i , GTSP is the same as Traveling Salesman Problem. The GTSP can be reduced to a TSP for arbitrary $|V_i|$ using the reduction described in [110]. WLOG, assuming that graph G is a digraph and the partitions are numbered in such a way that $|V_i| \geq 2$ for $1 \leq i \leq r$ and $|V_i| = 1$ for $r + 1 \leq i \leq k$. For any $i \leq r$ let $V_i = \{i_1, i_2, \dots, i_{n_i}\}$. For each arc $e \in E$, allow a new cost d_e defined as follows:

$$d_{i_j i_{j+1}} = -M, j = 1, \dots, n_i \text{ with } n_i + 1 \equiv 1, i = 1, \dots, r. \quad (2.6)$$

This guarantee that if a minimal TSP tour in graph G enters a cluster V_i through node i_j , it visits nodes of V_i in the order $i_j, i_{j+1}, \dots, i_{n_i}, i_1, \dots, i_{j-1}$ and leaves the cluster V_i from the vertex i_{j-1} . Now the goal is to interpret this outcome to be equivalent to a GTSP tour entering and leaving the cluster V_i by visiting vertex i_j . To demonstrate this fact, the new cost of arcs which are going out of i_{j-1} must coincide with the original cost of arcs leaving i_j . In the following Equation 2.7:

$$d_{i_{j-1} p} = c_{i_j p}, p \notin V_i, j = 1, 2, \dots, n_i \text{ with index } 0 \equiv n_i, i = 1, \dots, r \quad (2.7)$$

and $d_{uv} = c_{uv}$ for all other edges. From a minimal solution of the TSP on graph G with the modified costs d_e for $e \in E$, a minimal solution to GTSP can be recovered. For more details on this problem, we refer to [62, 79].

The MAX TSP: In contrast to the TSP, the objective in the MAX TSP is to find a tour in graph G where the total cost of edges of the tour is maximum. This problem can be solved as a TSP by just replacing each edge cost by its additive inverse. If problem requires that the edge costs are non-negative, a large constant could be added to each of the edge-costs. These replacing will not change the optimal solutions of the problem. The MAX TSP has been discussed in [63].

Traveling Tourist Problem: A tourist wishes to see all monuments in a city, and so must visit each monument or a neighbour thereof. Furthermore, it is assumed that a monument is visible from any of its neighbours. The edges therefore represent lines of a sight. The resulting walk will therefore visit a subset of all nodes in the graph G . For the Traveling Tourist Problem we refer to the [95].

The bottleneck TSP: In this variation of TSP the objective is to find a tour in graph G such that the largest cost of edges in the tour is as small as possible. A bottleneck TSP can be described as a TSP with exponentially large edge costs. For more details we refer to [63].

TSP with multiple visits (TSPM): In this problem the objective is to find a routing of a traveling salesman. Salesman starts at a given vertex of graph G , visits each vertex at least once and comes back to the starting vertex in such a way that the total distance traveled is minimized. The TSPM can be transformed into a TSP

by replacing the edge costs with the shortest path distances in graph G . In the lack of negative cycles, shortest path distances between all pairs of vertices of a graph G can be computed using efficient algorithms [1]. If graph G contains a negative cycle, then TSPM is unbounded. More details on TSPM can be found in [63].

2.1.5 All-Pairs Shortest Paths

In this section, the problem of finding shortest paths between all pairs of vertices of graph is considered. Given a weighted, directed graph $G = (V, E)$ with a weight function $\omega : E \rightarrow \mathbb{R}$ that maps edges to real valued weights. The objective is to find, for every pair of vertices $u, v \in V$, the shortest path from u to v . In addition the weight of a path is the sum of the weights of its edges. The all-pairs shortest-paths problem, can be solved, by running a single-source shortest-paths algorithm $|V|$ times, once for each node as the source. In some particular case, where all edge weights are nonnegative, *Dijkstra's algorithm* [28] can be used for solving the problem. If the linear-array implementation of the min-priority queue is used, then the running time is $O(V^3 + VE) = O(V^3)$. Furthermore, the binary min-heap implementation of the min-priority queue generate a running time of $O(VE \lg V)$, which is an improvement, if the graph is *sparse*. If the graph contains the negative-weight edges, then Dijkstra's algorithm can not be used. Alternatively, the Bellman-Ford algorithm can be run, once from each node. The resulting running time is then $O(V^2E)$, which results to $O(V^4)$ on a *dense* graph. In addition, for solving the all-pairs shortest paths problem on sparse graphs the *Johnson's algorithm* is used. For more details see [28]. In contrast to the single-source algorithms, which assume a distance-list representation of the graph, the *Floyd-Warshall algorithm* uses a distance-matrix representation.

The Floyd-Warshall Algorithm

The Floyd-Warshall algorithm (FW) is a simple and extensively used algorithm for computing the shortest paths between all pairs of vertices in an edge weighted directed graph $G = (V, E)$. The algorithm runs in $O(V^3)$. The Floyd-Warshall algorithm produce the correct result as long as no negative cycles exist in the input graph. The FW algorithm take into consideration the intermediate nodes of a shortest path, where an intermediate node of a simple path $p = \langle v_1, v_2, \dots, v_l \rangle$ is any node of p other than v_1 or v_l , that is, any node in the set $\{v_2, v_3, \dots, v_{l-1}\}$. The Floyd-Warshall algorithm depend on the following observation. During assumption that the nodes of G are $V = \{1, 2, \dots, n\}$, consider a subset $\{1, 2, \dots, k\}$ of nodes for some k . For any pair of nodes $i, j \in V$, consider all paths from i to j whose intermediate nodes are all drawn from $\{1, 2, \dots, k\}$, and let p be a minimal-weighted path between them. The Floyd-Warshall algorithm take advantage of a relationship between path p and shortest paths from i to j with all intermediate nodes in the set $\{1, 2, \dots, k-1\}$. Furthermore, this relationship depends on whether or not k is an intermediate node of path p .

The running time of the Floyd-Warshall algorithm shown in Algorithm 1 is appointed by the triply nested **for** loops of lines 2-4. Furthermore, each execution of

Algorithm 1 Floyd-Warshall

```
1: procedure FLOYD-WARSHALL( $V, W$ )
2:   for all  $k \in V$  do
3:     for all  $i \in V$  do
4:       for all  $j \in V$  do
5:          $W_{ij} := \min(W_{ij}, W_{ik} + W_{kj})$ 
6:       end for
7:     end for
8:   end for
9: end procedure
```

line 5 takes $O(1)$ time, so the algorithm runs in time $O(V^3)$. Because of that the Floyd-Warshall algorithm is pretty efficient for even modest sized input graphs.

2.2 Algorithms for the Traveling Salesman Problem

In this section, different algorithms that have been recommended in the literature for the Traveling Salesman Problem will be examined. This section introduces the basic concepts of heuristic approaches for the TSP and give some of the theoretical results that have been examined in the literature. Heuristic approaches correspond to approximation algorithms with objective to find the near optimal solutions quickly rather than the best solution to a given problem. The scope of the section mainly concentrates on the classical constructive, local search approaches and by nature inspired techniques and their extensions. For the TSP these diverse approaches represent the state-of-the-art approaches when the objective is to find satisfactory solutions quickly. Heuristic approaches for the TSP have been presented in several surveys. For the comprehensive overview of the heuristic techniques, we refer a reader to the surveys [74, 78] and to a book on the TSP and its variants [63].

2.2.1 Tour Quality

The classic measure of the quality of a tour length is the gap to the optimal tour length. This gap is commonly demonstrated as the pass over optimum. For algorithm A this gap can be formally shown as relative difference between obtained length of a tour l_A and optimal length of a tour l_C :

$$\frac{l_A - l_C}{l_C}. \quad (2.8)$$

Furthermore, this gap is always expressed in percents. Luckily, the standard test beds, such as Reinelt's TSPLIB [122], provides a wide set of instances, which also includes the optimal tour length for all test instances. The optimal tour lengths are usually unknown for instances of sizes or structure that cannot be solved to optimality in a reasonable amount of time. Because of the lack of the optimal tour length for instances, a different reference point is needed. Those reference points present values,

which are close to the optimal tour length. Furthermore, this reference points are as a consequence commonly a lower bound on the optimal solution.

The Held-Karp lower bound (HKLB) [69] represents a standard lower bound for the TSP. The HKLB on the optimal solution corresponds to the solution of a linear programming relaxation [23] of the standard integer programming formulation of the TSP. This imply that the integer constraints of the integer programming problem are substitute with bounded variables. Furthermore, the resulting linear programming problem [132] is then solved to optimality by using an algorithm such as the simplex method [32]. Even if the resulting linear program consist of an exponential number of subtour constraints it can be solved in polynomial time, because there exists a polynomial time "seperation oracle" for the subtour constraints based on calls to a max-flow algorithm [80]. Experimental results, which is presented in [63], show that this lower bound is very close to the optimal tour length in most cases.

2.2.2 Tour Construction Algorithms

The objective of tour construction approaches is to build a tour for an instance of traveling salesman problem from scratch. This can be achieved by constructing a tour which will follow some construction rule. Determinations during the construction of a tour are mostly made in a greedy fashion. The objective of these constructive methods is an immediate gain instead of looking ahead. There are many constructive heuristics which have been suggested for the TSP. A comprehensive overview of tour construction approaches for the TSP can be found in [10,123]. Furthermore, for the thorough performance testing of the tour construction approaches for the symmetric TSP, especially for the Euclidean TSP, see [63,123].

The tour construction approaches described in the next two subsections were chosen because of their suitability to symmetric, especially Euclidean instances, of Traveling Salesman Problem. Two basic and from the dissertation point of view important tour construction approaches are the *Nearest Neighbour* and the *Insertion* heuristics.

Nearest Neighbour Heuristic

One of the most intuitive heuristic algorithm for the TSP is the Nearest Neighbour algorithm (NN). The salesman starts at an arbitrary selected node and then sequentially move to the nearest node he has not yet visited. When all nodes have been visited, salesman returns to the node he started from. The pseudocode of this algorithm for a set of nodes, i.e. , cities $C = \{c_1, c_2, \dots, c_n\}$ is shown in Algorithm 2. The NN algorithm has a computational complexity of $O(n^2)$ which can be reduced to $O(n \log n)$ for geometric instances [10]. The best performance ratio noted for the instances of TSP, which satisfy the triangle inequality is given by $\frac{NN(i)}{OPT(i)} \leq 0.5 (\lfloor \log_2 n \rfloor + 1)$, see [74], and therefore growing in n . Nevertheless, in another case where the triangle inequality is not satisfied one can anticipate even worse the performance ratio. In the literature exists various variants of the NN approach

with objective to improve its performance. One such variant is the Double Ended Nearest Neighbour (DENN) [123], where the tour is built from both ends of the current sequence. Another variant is so-called Random Nearest Neighbour (RNN), where the node, which will be visited next is selected randomly from a set of nearest neighbours [63].

Algorithm 2 Nearest Neighbour

```
1: procedure NEARESTNEIGHBOUR( $C$ )
2:   Select arbitrary city  $c_j$ , set  $k = j$  and  $C = \{c_1, \dots, c_n\} \setminus c_j$ 
3:   while ( $C \neq 0$ ) do
4:     Determine  $c_l \in C$  with  $d(c_k, c_l) = \min_{c_j \in C} (d(c_k, c_j))$ 
5:     Add  $c_l$  to the tour by connecting  $c_k$  to  $c_l$  and set  $c \leftarrow C \setminus c_l$  and  $k = l$ 
6:   end while
7:   Connect  $c_k$  to starting city  $c_j$ 
8: end procedure
```

Insertion Heuristic

For the insertion heuristics a different construction rule is followed. The algorithm start with a subtour, which contains one or two cities. Then it successively add cities to the current subtour. This adding is followed by some selection criteria. The pseudocode of this approach is presented in Algorithm 3.

Algorithm 3 Insertion

```
1: procedure INSERTION
2:   Select a starting tour through  $k$  cities  $T = \{c_1, c_2, \dots, c_k\}$ 
3:   repeat
4:     Select a city  $c_i$  with  $c_i \notin T$  following some criteria
5:     Insert city  $c_i$  into the current subtour  $T$ 
6:   until  $c_i \notin T, i = 1, 2, \dots, n$ 
7: end procedure
```

From the description of the algorithm three questions arise: From which city or cities to start the heuristic? Which city to insert next? Where to insert the city into the current subtour? Various decision rules have been proposed in the literature regarding the answers on these questions. The set of cities, which construct the starting subtour is commonly selected randomly and consists of one, two or three cities. For Euclidean problems other approaches such as starting the heuristic from the convex hull of the problem have been also suggested [123]. The last two questions, to which city to insert next and where to insert it, are tightly related. Some selection rules are:

1. Random Insertion: Choose the city to be inserted randomly from the set of cities that are not yet a component of the tour,
2. Cheapest Insertion: Insert the city, which is not yet a component of the tour,

whose insertion results to the minimal grow in tour length,

3. Farthest Insertion: Insert the city, which is not yet a component of the tour, whose minimal distance to a city, is maximal,

4. Nearest Insertion: Insert the city which is not yet component of the tour and is nearest to any city which is at present a component of the tour. A comprehensive empirical analysis of Insertion Heuristics with different insertion criteria can be found in [123]. Nearest insertion, farthest insertion and random insertion can be implemented to run in $O(n^2)$ time. Cheapest insertion has time complexity $O(n^2 \log n)$ and is computationally more expensive.

2.2.3 Local Search Algorithms

Local search algorithms for the Traveling Salesman Problem are built on simple tour adjustments. A local search algorithm is constructed from operations called moves which are used to transform one tour to another. The local search is actually a neighborhood search process where each tour has an associated neighborhood of tours. The local search algorithm repeatedly moves to a better neighbor as far as no better neighbors exist. These moves which have been proposed for the TSP can be generally divided into operators of node exchange, node insertion, and edge exchange. The node exchange operator work in such a way that it exchanges two nodes in the sequence. On the other hand the node insertion operators work by deleting a node from a tour and inserting it at another position in the tour. The edge exchange operators exchange the edges in the tour and will be described in detail in after section. For the example of the TSP a tour t^* , is called locally optimal when all other tours in its neighbourhood are at least as long as t^* . To identify a local optimum, the neighbourhood of an initial tour is searched for a solution of better quality. When a new best solution s' , has been found, it is accepted and its neighbourhood $N(s')$ is explored. This process is repeated as far as no more improvements can be found. A pseudocode using above notifications of solutions and neighbourhoods is given in Algorithm 4.

Algorithm 4 Local Search

```
1: procedure LOCALSEARCH
2:   Create feasible Solution  $s$ 
3:   Choose a neighbourhood function  $N$ 
4:   repeat
5:     Search neighbourhood  $N(s)$ 
6:     if ( $s'$  with  $f(s') < f(s)$  found) then
7:       Set  $s = s'$ 
8:     end if
9:   until no feasible lower cost solution  $s'$  is found
10: end procedure
```

The objective of local search algorithm is to consecutively apply improving moves to the solution in order to explore the neighbourhood introduced by N . For the TSP

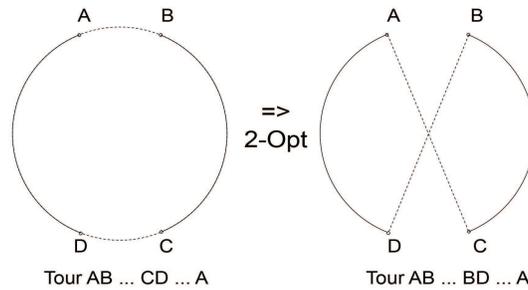


Figure 2.1: Edge removal and reconnection of 2-opt algorithm

this neighbourhood is commonly based on simple tour modifications.

***k*-opt Local Search**

Amid different local search algorithms, the most studied are *k*-exchange neighbourhood which is an example of edge exchange algorithm. The *k*-exchange neighbourhood is for the Traveling Salesman Problem commonly referred to as *k*-opt. For the TSP this local search can be defined as follows. Let S corresponds to the set of all tours of a TSP instance. Let us also introduce a metric p between tours in S , which measures the distance between tours with the number of edges not mutual to both, i.e. the number of edges which represent part of tour T but not part of tour T' . The *k*-opt local search can then be stated by:

$$N_k(T) = \{T' | p(T, T') = k, T' \in S\}.$$

From the above definition the reader can see that this statement introduces a whole set of search neighbourhoods which is parameterised by k . Usually, the higher the k of a *k*-opt local search, the better the resulting tours. In spite of this, since the neighborhood size grows exponentially with k , only small k appear to be practical.

2-opt

The 2-opt local search illustrates the simplest of the *k*-opt local search algorithms for the TSP. The 2-opt algorithm was first proposed in [30], even though the basic move had already been suggested in [49]. This move deletes two edges, as a consequence the tour is aperted into two segments, and then algorithm reconnects those segments in the other possible way. A schematic illustration of this edge removal and reconnection approach is presented in Figure 2.1.

Just as is shown in the figure, this operation is equal to a tour segment reversal. The order of the tour segment B, \dots, C is reversed with respect to C, \dots, B . For the Euclidean TSP, 2-opt local search removes the *crossings* of edges in the tour. For symmetric TSPs a 2-opt move usually produce an improvement to the current tour if $d(A, C) + d(B, D) < d(A, B) + d(C, D)$. There exist two different representations

of a 2-opt move, i.e. the move presented in the Figure 2.1 is equivalent to the move where the tour segment between D and A is reversed. Since for a 2-opt move to denote an improvement either $d(A, B) > d(B, D)$ or $d(C, D) > d(A, C)$ or both must hold. Because of this the attention can be concentrate on moves satisfying $d(A, B) > d(B, D)$. This imply that once A and B are fixed we can limit the search for an improving move on cities D that are closer to B than A .

So to take advantage of this feature Steiglitz and Weiner [128] introduce a data structure to specify suitable candidates quickly. And so, storing for each city a list of the remaining cities sorted with increasing distances. When seeking for a 2-opt move, one has to start at the beginning of the list of B , move forward through the list observing its members until $d(A, B) \leq d(B, D)$. The difficulty of this approach is that creating the lists is of time complexity $\Theta(n^2 \log n)$ and requires space quadratic in n . As a consequence in practice this list is commonly restricted to only k -nearest neighbours for each city with fixed k . Even if the resulting tour may not be locally 2-optimal, as not all possible moves are considered. In the practice, in general, only a small loss of tour quality is reported [74].

The Lin-Kernighan Algorithm

For over 30 years the world's champion heuristic for the Traveling Salesman Problem was usually recognized to be the local search algorithm of Lin and Kernighan (LK) [96]. This algorithm is both a generalization of the 2-opt local search algorithm and an outgrowth of ideas the same authors had previously applied to the graph partitioning problem. The fundamental idea of the LK heuristic is to build up complex moves by combining simple submoves to replace a variable number of edges. Because of this property this tehniqe is also called a variable depth k -opt. The submoves commonly employed are 2-opt and 3-opt moves. The LK heuristic can be described as follows. In each step, the tour is broken up at one node forming a 1-tree (a spanning tree with an extra edge). This 1-tree can be with ease transformed into a possible TSP tour by breaking up one edge of the degree-3 node and connecting the two degree-1 nodes. Consequently the heuristic performs sequential changes of edges until no further exchanges are possible or until the best k -change in an iteration is found. A more comprehensive description of the LK algorithm would go way beyond the scope of a dissertation and can be found in the paper by Lin and Kernighan [96]. The original Lin-Kernighan heuristic was suggested for the symmetric TSP only.

A main drawback of the LK algorithm, besides the high effort needed for its implementation, is its rather long running time. Because of this, several improvements to the original algorithm have been made. In the paper [123] Reinelt, describes a variant of LK algorithm using the segment reversals and node insertion. In this variant a special 3-opt move consists of a single city to enlarge the Lin-Kernighan neighbourhood. Some different approaches can be found in [138], and in [120]. In this two papers the search neighbourhood is the so called flower transition, firstly suggested by Glover in [56] where the base is a 1-tree consisting of a cycle and a path attached to it. A effective variant of LK heuristic based on a sequence of 5-opt moves instead of 2-opt segment reversals was suggested in 2000 by Helsgaun [70].

The estimation of these move sequences is accomplished by considering very small candidate lists which are determined in a very complex manner. Other approaches and variants of the LK heuristic have try to beat problems produced by very large TSP instances. Otherwise, to make the Lin-Kernighan searches perform better on the problematic instances of TSP [6]. Even though the variant of LK heuristic proposed by Helsgaun [70] is commonly the most successful one, when only a tour quality is considered. Otherwise, it is hard to say which one of these variants is the most effective. For a comparison of the Lin-Kernighan implementations we refer a reader of a dissertation to a Johnson and McGeoch work on heuristics for the symmetric Traveling Salesman Problem [75].

2.2.4 Nature Inspired Algorithms

The area of nature-inspired computing has grown in popularity over the last fifty years. Many of the nature-inspired algorithms currently in use are being applied to a wide range of problems, among them are of course ones tackling the Traveling Salesman Problem. The term *nature* is used to refer to any component of the universe which is not a product of planned human design. The nature-inspired algorithms are in the category of the metaheuristic algorithms, where little or no problem particular information is used in the design of the algorithm. Beyond all doubt the most important contribution to biology was made by Charles Darwin with his Theory of Evolution by Natural Selection. As Chiong comments in [22] Observing the achievements of animals, Darwin writes:

Why, if man can by patience select variations most useful to himself, should nature fail in selecting variations useful, under changing conditions of life, to her living products.

If we restrict our attention to the biological part of nature, we can highlight some of the useful properties. In nature, managing the trade-off between solution quality and time is basic to survival. A likewise trade-off is made when using a heuristic to solve the optimization problems. The fittest individuals are those with supreme problem-solving feature. It is these problem-solving features which have been the source of inspiration for many nature-inspired techniques. We will now represent several metaheuristic examples of nature-inspired algorithms for optimization problems, above all those techniques which is used for tackling the Traveling Salesman Problem.

Simulated Annealing

Simulated Annealing (SA) was introduced by Kirkpatrick et.al. in [85]. The SA optimization method builds on a similarity derived from physics processes where a low energy state of a solid is inquired by an annealing procedure. The similarity with combinatorial optimization occur when the optimal solution to a given combinatorial optimization problem corresponds with the lowest energy of the solid. Therefore, a

solution to a problem is somehow perturbed and a neighbor solution is accepted with probability according to a Boltzmann distribution $e^{\frac{-\Delta E}{k * T}}$. Here ΔE correspond to the difference in quality between the current solution and the perturbed one. Furthermore, k is a scaling parameter and T correspond to the temperature of the process. The higher the temperature the greater is the probability of acquiring a perturbed solution. When the temperature is low, improving moves will be privileged. By reducing T using an "annealing schedule" it is possible to simulate the freezing process. The pseudocode of the SA is shown in Algorithm 5:

Algorithm 5 Simulated Annealing

```
1: procedure SIMULATEDANNEALING
2:    $t = T(0)$ ,  $n = 1$ 
3:   best solution  $s_{best} = S$ 
4:   while (Termination Criterion Unfulfilled) do
5:     Generate  $s' \in N(s)$ 
6:     /* we obtain a neighbor of  $s$  using the move operator  $N()$  */
7:      $\Delta f = f(s) - f(s')$ 
8:     /* we compute the difference in fitness */
9:     if ( $(\Delta f \leq 0)$  or ( $e^{\frac{-\Delta E}{k * T}} > random [0, 1]$ )) then
10:       $s = s'$ 
11:      /* we accept the perturbed move if it is better than the current */
12:      /* or if the Boltzmann criterion is satisfied */
13:    end if
14:    if ( $f(s) > f(s_{best})$ ) then
15:       $s_{best} = s$ 
16:    end if
17:     $t = T(n)$ 
18:    /* we apply the annealing schedule for time  $n$  */
19:     $n = n + 1$ 
20:  end while
21:  Return  $s_{best}$ 
22: end procedure
```

The key to the efficiency of SA in solving a specific combinatorial optimization problem stands in the definition of the the annealing schedule and move operator. Since the first appearance of SA, the Traveling Salesman Problem has served as a test problem, considered in the original paper of Kirpatrick [85]. For these particular approach the SA neighbourhood was 2-opt local searcher. In addition, Boese's PhD dissertation [13] comprehensively analyzes the optimal annealing schemes for the TSP.

Ant Colony Optimization

A recent and increasingly popular metaheuristic approach inspired by nature is the Ant Colony Optimization (ACO) or Ant Algorithms introduced in 1997 by

Dorigo. ACO [39] presents a population based approach that was inspired by the behavior of real ants in nature. The main idea of this approach trace the estimation that ants in nature appear very effective in finding the shortest paths to a food source. The ants are capable to find the shortest paths through communication transmitted by leaving pheromone trails while exploring for food. For the Traveling Salesman Problem, ACO represents a constructive approach which is frequently updated until some stopping condition is reached. Starting with n ants, each located at a different city, the ants consecutively move along the edges to create feasible tours. Determination on which city to visit next are made in a probabalistic manner based on the pheromone trail left at earlier investigation. After each iteration the pheromone trail is updated depositing a higher amount of pheromone at edges used in the shortest tours.

Several improvements to this basic approach have been proposed in the literature, especially the conjunction with local search to speed up the search process [129]. The computational results described in the literature range from very poor to moderate quality in [40] to solutions of very good quality presented in [129]. For more knowledge on ACO for the TSP and also to some other combinatorial optimization problems we refer to [40, 129].

Evolutionary Algorithms

Evolutionary Algorithms (EA) simulate the process of biological evolution in nature. These are search methods which are motivated by natural selection and survival of the fittest from biological world. EA performs a search using a population of solutions. Each iteration i.e. generation of an EA includes a competitive selection among all solutions in the population. This selection results in survival of the fittest and erasure of the poor solutions from the population. Recombination is performed by exchanging parts of a solution with another one. This proces forms the new solution that may be better than the previous ones. Furthermore, a solution can be mutated by manipulating a part of it. The evolutionary operators, recombination and mutation, are used to evolve the population towards areas of the space in which good solutions exist. Four major evolutionary algorithm classes have been introduced during the last 50 years: genetic programming is a computational method, which was proposed Koza [86], Evolutionary Strategies (ES) developed by Rechenberg [118], Evolutionary Programming (EP) introduced by Fogel [50], and Genetic Algorithms (GA), proposed by Holland [71]. The general template of an EA is shown in Algorithm 6. All mentioned variants of evolutionary algorithm (GA, EP, and ES) are special cases of this scheme.

For the Traveling Salesman Problem the evolutionary algorithms may be shortly outlined as follows. Starting from a population of k individuals i.e. tours, select k' different individuals for mutation and parents for mating. Perform mutations on the selected individuals and the mating recombination. In recombination the information of two parent individuals are combined in order to create an offspring individual. Then select k existing individuals of the new population succeeding the selection strategy. This procedure is consecutively applied for some iterations i.e.

Algorithm 6 Evolutionary Algorithms

```
1: procedure EA
2:    $t := 0$ 
3:   initialize population ( $P(0)$ )
4:   evaluate ( $P(0)$ )
5:   repeat
6:      $P' :=$  select for variation ( $P(t)$ )
7:     recombine ( $P'$ )
8:     mutate ( $P'$ )
9:     evaluate ( $P'$ )
10:     $P(t + 1) :=$  select for survival ( $P(t), P'$ )
11:     $t := t + 1$ 
12:  until terminate = true
13: end procedure
```

generations until some termination criteria is met. For further reading on Evolutionary Algorithms we refer to [7, 106, 127].

Genetic Algorithms

Genetic Algorithms were introduced by Holland in the 1970s [71], and well researched by many authors due present date [43, 51, 52, 103, 109, 117, 126, 130, 134, 135, 139, 140]. These techniques are adaptive search methods based on the instrument of natural selection and the survival of the fittest concept. A comprehensive introduction to Genetic Algorithms is given in Goldberg's book [58]. The main inspiration behind GA is to start with randomly created initial solutions and implement the survival of the fittest strategy to develop the better solutions through iterations i.e. generations. A Genetic Algorithm process includes initial population generation, fitness evaluation, chromosome selection and applying the genetic operators for reproduction, recombination and mutation. The pseudocode of a GA can be seen in Algorithm 7.

Algorithm 7 Genetic Algorithm

```
1: procedure GA
2:   Randomly generate an initial population  $P(t)$ 
3:   while (Termination Criterion Unfulfilled) do
4:     Compute the fitness  $f(p) \forall p \in P(t)$ 
5:     According to  $f(p)$  choose a subset of  $P(t)$ , store them in  $M(t)$ 
6:     Recombine and mutate individuals in  $M(t)$ , store results in  $M'(t)$ 
7:     Generate  $P(t + 1)$  by selecting some individuals from  $P(t)$  and  $M'(t)$ 
8:      $t = t + 1$ 
9:   end while
10:  Return best  $p \in P(t - 1)$ 
11: end procedure
```

In designing a Genetic Algorithm, how to encode a search solution is a basic and key issue [21]. Many optimization operators for Traveling Salesman Problem were proposed by Goldberg [58]. A usually used encoding strategy is transposition expression [119]. In the transposition expression encoding strategy, each city of the TSP is encoded as a gene of the chromosome. This encoding include the constraint that each city appears once and only once in the chromosome. Transposition expression is the most nature expression for TSP which based on the order of tour, on the other hand such a procedure may leads to infeasible tour after traditional crossover operator. This is a usual occurrence for TSP. Even if feasibility can be maintained in many ways by some repair algorithms, such algorithms can spend a substantial amount of time and can inhibit convergence [119].

To overcome a possible creation of infeasible tours another encoding method was introduced. It is the Random Keys encoding [9] which is introduced by Bean. In random keys encoding a random numbers encode the construction of the solution. Such representation guarantees that feasible tours are preserved during the application of genetic operators. In the GA, the crossover and mutation are two of most important factors for the success of the algorithm. Crossover is a recombining operator that takes two individuals, and cuts their chromosome strings at a number of chosen positions. The sub-segments produced by that cuts are then mixed and reconnected to produce a whole chromosome with features of the two parents. Therefore, the basic role of recombination is of information exchange between successful solutions.

Numbers of different recombination operators have been proposed in the literature to solve the TSP using a GA. The partially mapped crossover [58], linear order crossover [34] and order based crossover [9, 58] are the usually used recombination techniques in the TSP context. Except the usually used recombination strategy, many different recombination operators are proposed for the TSP, for example: sub-tour crossover [137], edge map crossover [51], distance preserving crossover [103], generic crossover [104], NGA [77], EAX [107], GSX [108], heuristic based crossover [94].

The other genetic operator usually used is mutation. It is used to produce the variation of genomes. Mutation is applied stochastically to a child after recombination. It alters one or more genes with a small probability allowing a small amount of random search. As a result of that no point in the whole search space has a zero probability of being visited by the genetic algorithm. Therefore, a mutation operator is used to enhance the diversity and provide a chance to escape from local optima. Many mutation operators were proposed such as inverse, insert, displace, swap, hybrid mutation [82], and heuristic mutation. The first five listed here are realized by small modification of genes. Heuristic mutation was proposed by Cheng and Gen [20], this operator adopts a neighborhood strategy to improve the solution.

Overall the Genetic Algorithms have shown themselves to be useful and efficient when the search space is large, complex or poorly understood. A lot of progress was made recently. In 2006, Carter and Ragsdale propose a new GA chromosome and to them related operators for the Multiple TSP [19]. In 2007, Nguyen described a hybrid GA based on a parallel implementation of a multi population steady-state GA involving local search heuristics [109].

Edge Map Crossover

Edge Map Crossover (EMX) [51] is an implementation of the recombination operator for Genetic Algorithms. It makes use of a so called edge map. Edge map is a table in which each location is placed. For each location there is a list in which the neighbouring locations are listed with this location. Recombination is then established as follows. Choose the first location of one of both parents to be the current location. Second step is to remove the current location from the edge map lists. If the current location still has remaining edges, go to the previous step, otherwise go to the next step. Choose the new current location from the edge map lists of the current location as the one with the shortest edge map list. If there are remaining locations, choose the one with the shortest edge map list to be the current location and return to second step.

Example:

Parents: 1-2-3-4-5-6; 2-4-3-1-5-6

Edge map: 1) 2 6 3 5; 2) 1 3 4 6; 3) 2 4 1;
4) 3 5 2; 5) 4 6 1; 6) 1 5 2 6

1. Random choice: 2,
2. Next candidates: 1 3 4 6, choose from 3 4 6 same#edges, choose 3,
3. Next candidates: 1 4 (edge list 4 < edge list 1), choose 4,
4. Next candidate: 5, choose 5,
5. Next candidate: 1 6 (tie breaking) choose 1,
6. Next candidate; 6, choose 6.

Offspring: 2-3-4-5-1-6

Distance Preserving Crossover

Distance Preserving Crossover (DPX) is another implementation of the recombination operator for Genetic Algorithms. It attempts to create a new tour with the same distance to both parents [70]. In order to establish this, the content of the first parent is copied to the offspring and all edges that do not occur in the second parent are removed. The resulting fragments are reconnected without making use of non-overlapping edges of the parents. If edge (i, j) has been destroyed, the nearest available neighbor k of i from the remaining fragments, is selected and the edge (i, k) is added to the tour.

Example: Parents: 5-3-9-1-2-8-0-6-7-4; 1-2-5-3-9-4-8-6-0-7

Fragments: 5-3-9|1-2|8|0-6|7|4

Offspring: 6-0-5-3-9-8-7-2-1-4

We proceed to next subsection which will in more details describe the hybrid genetic algorithms.

Hybrid Genetic Algorithms - Memetic Algorithms

Hybrid Genetic Algorithms are evolutionary algorithms that include a stage of individual optimization or learning as part of their search strategy. Some, but not all, of the references can be traced to [48, 68, 133]. The most basic Hybrid Genetic Algorithm can be seen below in Algorithm 8 :

Algorithm 8 Hybrid Genetic Algorithm

```
1: procedure HYBRIDGA
2:   Randomly generate an initial population  $P(t)$ 
3:   while (Termination Criterion Unfulfilled) do
4:     Compute the fitness  $f(p) \forall p \in P(t)$ 
5:     According to  $f(p)$  choose a subset of  $P(t)$ , store them in  $M(t)$ 
6:     Recombine and mutate individuals in  $M(t)$ , store results in  $M'(t)$ 
7:     Improve by local search ( $M'(t)$ )
8:     Generate  $P(t+1)$  by selecting some individuals from  $P(t)$  and  $M'(t)$ 
9:      $t = t + 1$ 
10:  end while
11:  Return best  $p \in P(t-1)$ 
12: end procedure
```

In the pseudocode of the hybrid genetic algorithm the difference feature from a standard GA is the use of local search. It is used to improve the newly created individuals. The reader should note that this is just one possible way to hybridize a GA with local search. Even though it seems from the Algorithm 8, to be a naive minor change, is in fact a crucial deviation from a canonical GA. Hybrid genetic algorithms are inspired by model of adjustment in natural systems that combine evolutionary adjustment of populations of individuals with individual learning.

In the literature, Hybrid Genetic Algorithms have also been named Memetic Algorithms (MA) [62, 87–89, 104, 113], Genetic Local Searchers (GLS) [102], Lamarckian Genetic Algorithms [105], Baldwinian Genetic Algorithms [90]. The Memetic Algorithms differ from other hybrid evolutionary techniques in that all individuals in the population are local optimum, since after each mutation or recombination, a local search is applied. The name genetic local search (GLS) was firstly used by Ulder in [131] to describe an evolutionary algorithm with recombination and then applied local search. In [17], Bui suggest a GLS algorithm with Lin-Kernighan heuristic as the neighbourhood procedure. They developed a k -point recombination operator with an additional repair mechanism for producing the feasible offspring. In [81] Katayama suggest an evolutionary algorithm with Lin-Kernighan algorithm and small populations, (he used just two individuals) and a heuristic recombination scheme. This approach is likewise to the iterated Lin-Kernighan heuristic but increased diversification is achieved by the crossover of the current solution and the best solution found.

In Chapter 3 we will systematically study approaches to Hybrid Genetic Algorithm. Furthermore, we are going to investigate some of the constructive and local

search approaches we have described in this chapter and to compare their performance to our grafted genetic heuristic approach for symmetric Traveling Salesman Problem.

2.2.5 Finding exact solutions for the TSP

Finding the exact solution to a Traveling Salesman Problem with n cities involves to check $(n-1)!$ of possible tours. Evaluation of all possible tours is infeasible for even small instances of TSP. For finding the optimal tour Held and Karp [69] introduced the following dynamic programming formulation: Given a subset of city pointers, discarding the first city, $S \subset \{2, 3, \dots, n\}$ and $l \in S$, let $d^*(S, l)$ stand for the length of the shortest path from city 1 to city l , visiting all cities in S in between. For $S = \{l\}$, $d^*(S, l)$ is defined as d_{1l} . Then the shortest path for larger sets with $|S| > 1$ is:

$$d^*(S, l) = \min_{m \in S \setminus \{l\}} (d^*(S \setminus \{l\}, m) + d_{ml}). \quad (2.9)$$

In conclusion, the minimal tour length for a complete tour which includes returning to city 1 is:

$$d^{**} = \min_{l \in \{2, 3, \dots, n\}} (d^*(\{2, 3, \dots, n\}, l) + d_{l1}). \quad (2.10)$$

Using the Equation 2.9 and the Equation 2.10, the quantities $d^*(S, l)$ can be calculated recursively and the minimal tour length d^{**} can be obtained. In a next step, the optimal permutation $\pi = \{1, i_2, i_3, \dots, i_n\}$ of city pointers 1 through n can be calculated oppositely, starting with i_n and working consecutively back to i_2 . The step take advantage of the fact that a permutation π can be optimal only if

$$d^{**} = d^*(\{2, 3, \dots, n\}, i_n) + d_{i_n 1} \quad (2.11)$$

and, for $2 \leq p \leq n-1$,

$$d^*(\{i_2, i_3, \dots, i_p, i_{p+1}\}, i_{p+1}) = d^*(\{i_2, i_3, \dots, i_p\}, i_p) + d_{i_p i_{p+1}} \quad (2.12)$$

The complexity of space for storing the values for all $d^*(S, l)$ is $(n-1)2^{n-2}$ which strictly restricts the dynamic programming algorithm to TSP of small sizes. In spite of that for very small TSP instances this approach is fast and efficient [66].

A quite different method can deal with larger instances by using a relaxation of the LP problem. This procedure iteratively tightens the relaxation till a solution is found. This method for solving LP problems is called *cutting plane method* and was introduced by Dantzig, Fulkerson, and Johnson in 1954 [31].

Each iteration of a method begins with using the relaxation $Ax \leq b$, where the polyhedron P determined by the relaxation contains S and is bounded. In addition the optimal solution x^* of the relaxed problem can be reached using standard LP solvers. If the x^* found belongs to S , the optimal solution of the original problem is obtained. If not then a linear inequality can be found which is content by all

points in S but violated by x^* . This inequality is called a cutting plane or cut. If no additional cutting planes can be found or the improvement becomes very small, the problem is branched into two subproblems. These sub-problems can be minimized individually. Branching is done iteratively that leads to a binary tree of subproblems. Then each of a subproblem is solved without further branching or is found to be irrelevant. Irrelevant means that relaxed version previously produces a longer path than a solution of another subproblem. This method is called branch and cut and was introduced by Padberg and Rinaldi, in 1990 [116]. The branch and cut method is a variation of the branch and bound procedure presented by Land and Doig, in 1960 [91].

The initial polyhedron P used by Dantzig [31] contains all vectors x such that for all $e \in E$ is $0 \leq x_e \leq 1$. Furthermore, in the resulting tour each city is connected to exactly two other cities. Different methods for finding cuts to prevent sub-tours (*sub-tour elimination inequalities*) and to ensure an integer solution *Gomory cuts*, were developed over time [66]. At present the most effective implementation of this method is *Concorde* described in [4]. *Concorde* is a computer code for the symmetric traveling salesman problem. The code is written in the *AnsiC* programming language. At the time of writing this dissertation the *Concorde*'s TSP solver has been used to obtain the optimal solutions to 106 of the 110 instances from TSPLIB [122]. In Chapter 3 the *Concorde*'s TSP solver was used for computing the lower bound for the quality of solutions for tested algorithms. Furthermore, in Chapter 4 it was used as a solver for the Traveling Salesman Problem.

Chapter 3

Grafted Genetic Algorithm for Traveling Salesman Problem

Results of this chapter are published in the following articles:

- M. Djordjevic, Influence of Grafting a Hybrid Searcher Into the Evolutionary Algorithm, *Proceedings of the Seventeenth International Electrotechnical and Computer Science Conference, ERK 2008.*, Portoroz, (2008), 115–118.
- M. Djordjevic, M. Tuba, and B. Djordjevic, Impact of Grafting a 2-opt Algorithm Based Local Searcher Into the Genetic Algorithm, *Proceedings of the 9th WSEAS International Conference on Applied Informatics and Communications, AIC 2009.*, Moscow, (2009), 485–490.
- M. Djordjevic, and A. Brodnik, Quantitative Analysis of Separate and Combined Performance of Local Searcher and Genetic Algorithm, *Book of Abstracts of International Conference on Operations Research, OR 2011.*, Zurich, (2011), 130.
- M. Djordjevic, and A. Brodnik, Quantitative Analysis of Separate and Combined Performance of Local Searcher and Genetic Algorithm, *Proceedings of the 33rd International Conference on Information Technology Interfaces, ITI 2011.*, Dubrovnik, (2011), 515–520.
- M. Djordjevic, M. Grgurovic, and A. Brodnik, Performance Analysis of Partial Use of Local Optimization Operator on Genetic Algorithm for Traveling Salesman Problem, *Business Systems Research*, Print ISSN 1847-8344; Online ISSN 1847-9375, in press.

3.1 Introduction

Genetic Algorithms (GA), which was in detail described in Section 2.2.4 use some mechanisms inspired by biological evolution [71]. They are applied on a finite set of individuals called population. Each individual in a population represents one of the feasible solutions of the search space. Mapping between genetic codes and the

search space is called encoding and can be binary or over some alphabet of higher cardinality. Good choice of encoding is a basic condition for successful application of a genetic algorithm. Each individual in the population is assigned a value called fitness. Fitness represents a relative indicator of quality of an individual compared to other individuals in the population. Selection operator chooses individuals from the current population and takes the ones, that are transferred to the next generation. Thereby, individuals with better fitness are more likely to survive in the population's next generation. The recombination operator combines parts of genetic code of the individuals (parents) into codes of new individuals (offsprings). Such a mixing of genetic material enables that well-fitted individuals or their relatively good genes give even better offspring. By a successive application of selection and crossover, the diversity of genetic material can be decreased, which leads to a premature convergence in a local optimum, which may be far from a global one.

The components of the genetic algorithm software system are: Genotype, Fitness function, Recombinator, Selector, Muter, Replacer, Terminator, and in our system a (Local) Optimizer which is a new extended component. In the TSP a set $\{c_1, c_2, \dots, c_n\}$ of cities is considered and for each pair $\{c_i, c_j\}$ of distinct cities a distance $d(c_i, c_j)$ is given. The goal is to find an ordering π of the cities that minimizes the quantity

$$\sum_{i=1}^{n-1} d(c_{\pi(i)}, c_{\pi(i+1)}) + d(c_{\pi(n)}, c_{\pi(1)}). \quad (3.1)$$

This quantity is referred to as the minimum tour length since it is the length of the tour a salesman would make, when visiting the cities in the order specified by the permutation, returning at the end to the initial city. We will concentrate in this chapter on the symmetric TSP, in which the distances satisfy $d(c_i, c_j) = d(c_j, c_i)$ for $1 \leq i, j \leq n$ and the distance is Euclidean. The TSP is known to be *NP-hard* [54], even under substantial restrictions, we are restricted to compute approximate solutions. Details about TSP can be found in Section 2.1.

There were proposed various greedy heuristics for solving TSP including nearest neighbor, details can be found in Section 2.2.2 and 2-opt. We will use the later in our algorithm as a hybridization component of a canonical GA. The 2-opt is a simple optimization algorithm for the TSP, it is in details described in Section 2.2.3. The main idea is to take a route that crosses itself and reorder it so, that it does not cross itself any more by reducing the tour length. An exchange step consists of removing two edges from the current tour and reconnecting the resulting two paths in the best possible way, as shown in Figure 2.1. Once we choose the two edges to delete, we do not have a choice about which edges to add. There is only one way to add new edges, that results in a valid tour.

The 2-opt optimization is used to hybridize GA meta-heuristic to solve TSP. Although the 2-opt algorithm [45, 70] performs well and can be applied to TSP with many cities, it finds only a local minimum. Furthermore, for the general (i.e. non-Euclidean) version of TSP it is known that there is no upper bound on a quality of a heuristic algorithms unless $P = NP$ [54]. Nevertheless, we will be interested in relative quality of our algorithm on a given set of samples.

In practice one of the most frequently used programs for solving TSP is Concorde [4]. It is based on the branch and cut method [66]. Besides it there is a range of solutions based on meta-heuristics including life-inspired approaches like ant colonies, see Section 2.2.4 and GA, see Section 2.2.4. Our algorithm is based on the later. Although there are known approaches that tried to combine canonical GA with local optimization (cf. Memetic Algorithms in [104], or hybridization of GA in [125], details can be found in Section 2.2.4), our solution is novel in two respects. First, we include local optimization as one of the operators of GA and secondly we do not use it in all generations.

The rest of the chapter is organized as follows. First we introduce the general framework of our algorithm including the principle of methodology and present the procedure of hybridization of TSP solving genetic algorithm. The third section describes two experiments and instances of TSP used in the experiments. The section is followed by experimental results, their analysis and discussion. The chapter concludes by summarizing the results, conclusion and suggestions for further research.

3.2 Grafted GA for the TSP

Grafting in botany is when the tissues of one plant are affixed to the tissues of another. Grafting can reduce the time to flowering, shorten the breeding program, etc. Similarly we introduced into a canonical GA a local optimizer - we grafted GA or we hybridized it. This way we (locally) optimize each genome in an evolution process. There exist a number of local optimizers, which can be used on their own as a greedy solution to NP-hard problems - e.g. Freisleben in [51] used a k-opt heuristics. There exists even its hardware implementation [72].

The pseudo-code of our Grafted Genetic Algorithm (GGA) is listed in Algorithm 9.

Algorithm 9 Grafted Genetic Algorithm

```
1: procedure GGA
2:    $t = 0$ 
3:    $p(t) := Initialize()$ 
4:    $q(t) := Evaluate(P(t))$ 
5:   while ( $q(t) < q_{expected}$ ) and ( $t < t_{max}$ ) do
6:      $sel := Select(P(t))$ 
7:      $mat := Mate(sel)$ 
8:      $rec := \mathbf{for\ each\ pair\ } m \in mat \mathbf{\ do\ } Recombine(m)$ 
9:      $loc := \mathbf{for\ each\ genome\ } r \in rec \mathbf{\ do\ } Optimize(r)$ 
10:     $P(t + 1) := Replace(loc, P(t))$ 
11:     $q(t + 1) := Evaluate(P(t + 1))$ 
12:    evaluate ( $P(t + 1)$ )
13:     $t := t + 1$ 
14:   end while
15: end procedure
```

As usual, our algorithm stops either when the expected quality of solution is reached or when the maximum number of generations t_{max} is passed. The former can be measured in various terms starting from the absolute quality value to the relative diversification of population (e.g. standard deviation). On the other hand the later is measured either in the number of generations or in the total time elapsed. Tournament Selector (line 6 in the algorithm) places groups of genomes from the population together, creating the groups from top to bottom with respect to the enumerative ordering of the genomes in the population and selects the best of the genomes within this group. This is repeated until the required amount of genomes is selected. The Random Mating (line 7 in the algorithm) is a simple way of mating parents. It mates the parents as enumerated in the population at random using the mating size to create groups until no more groups can be created. The new offspring only replacer is the implementation of the classical replacement strategy that simply only allows the offspring to survive. Thus the genomes from the next generation replace the entire current population.

We studied two versions of recombination (line 8 in the algorithm). The first, *Edge Map Crossover*, *EMX* for short [104], uses a so-called edge map $EM[*]$ that contains a list of neighbouring cities for each city. First the operator for each edge (u, v) in a parent genomes A and B , adds v to the edge-map list $EM[u]$ and u to $EM[v]$ for a symmetric version of TSP. Then the operator works as follows:

1. Pick a random city to be the current location u .
2. Remove the current location u from all edge map lists $EM[*]$.
3. If the current location $EM[u]$ still has remaining edges, go to step 4, otherwise go to step 5. item Choose the new current location u' from the edge map list $EM[u]$ as the one with the shortest edge map list $EM[u']$. Set $u := u'$ and go to step 2.
4. If there are left any locations, choose as the new current location u' the one with the shortest edge map list $EM[u']$. Set $u := u'$ and go to step 2.

The edge map crossover is in more details presented in Section 2.2.4. The second recombination operator, that was studied, was a *Distance Preserving Crossover*, *DPX* for short [51]. It creates a new tour (offspring) preserving the same distance in the number of edges to both parents. In detail, the operator DPX creates an offspring C from parents A and B as follows:

1. Pick at random one of the parents (*wlog.A*) and copy it to C .
2. If $(u, v) \in C$ and $(u, v) \notin B$ delete it from C . At this point C contains fragments of connected cities $(u_l \rightarrow v_l), \dots, (u_k \rightarrow v_k)$, where in some fragments $u_i = v_i$. We call set of u_i and v_i the end-points of C , EP_c .
3. Pick at random a city x from EP_c and delete it from EP_c .
4. Pick from EP_c the closest city y to x so that there is edge (x, y) neither in A nor in B . Delete y from EP_c .

5. Merge fragments $(u_i \rightarrow y)$ and $(x \rightarrow v_j)$ into $(u_i \rightarrow v_j)$.
6. If EP_c is not empty, go to step 3.

The distance preserving crossover is in more details described in Section 2.2.4. Although both recombination operators produced offsprings from valid parents, they produced them in a random way. Because of randomization, the selection and mating (lines 5 and 6 respectively) lost their role. Moreover, the randomization is a very good source of diversification. However, we are missing in our meta-heuristic the process of specialization.

This was the reason to extend our algorithm with a specialization step - we *grafted* a canonical genetic algorithm with a local optimizer and obtained a *grafted genetic algorithm*, *GGA* [35], [38], [36], [37]. We did not use a k-opt heuristics due to its complexity, but rather its simpler version 2-opt explained in previous section (see Figure 2.1). The hybridization occurs in line 9 of the pseudocode of Algorithm 9.

3.3 Experiment

For testing our strategy and comparing it to other solutions we used the instances of symmetric TSP found on TSPLIB [122]. We used relatively small instances, for which best solutions are known. The goal of this research was not to find a better algorithm, but rather to study on a controlled environment the impact of grafting a genetic algorithm.

In the first experiment we used 20 instances, with different sizes in a range from 14 to 150 cities per instance. We studied our proposed method (grafted genetic algorithm (GGA)) using two different recombination operators: an edge map crossover (GGAemx) and a distance preserving crossover (GGApx). As the upper and lower limits on the quality of solution we used greedy heuristic and Concorde [4] respectively. For the sake of completeness we compared our method also with 2-opt heuristic itself and with a canonical genetic algorithm. The main difference between our method and canonical genetic algorithm is that we use local optimizer in every generation of the algorithm.

In the second experiment we studied what happens if we do not use local optimization in all generations – in evaluation it was used in 10, 20, 30, 40, 50, 60, 70, 80 and 90 percents of the generations. Furthermore, for each percentage we applied local optimization in three different ways: at random generations, at the initial generations and at the ending ones.

All experiments were conducted on a computer with Pentium(R) 2.8 GHz CPU and Windows 7 operating system. In our results we cannot directly compare the running times of different solutions as they were implemented in different programming languages. On one hand we used as a development environment for GGA the Java written *EA Visualizer* [15], while *Concorde* is an *AnsiC* application. However, we can compare running times of GGA for different instances and cases explained before.

3.4 Evaluation

We present results separately for the first and for the second experiment. In both experiments we used instances of TSP from a *TSPLIB* of various sizes. The name of the instance also contains its size (the number of sites, cf. the first columns of Table 3.1). Next, in the experiments we measured three quantities: the wall clock time, the number of generations and the quality of the result. The later was measured against the optimal solution obtained by Concorde. The quality of algorithm *A* is defined as

$$q_A = \frac{l_A - l_C}{l_C} \quad (3.2)$$

where l_C is a path length obtained by Concorde and l_A is a path length obtained by *A*. We express the quality always in percents, where, for example, 4% means 4% worse than Concorde.

Let us look first at the results of the first experiment (cf. Table 3.1), in which we compared our GGA against greedy algorithm and Concorde. We also compared it against canonical genetic algorithm (GA). The termination condition in all genetic algorithms was: either standard deviation of genomes was 0 ((local) minimum was reached) or time 100 seconds time limit expired.

We first look at the quality of results, than at the running time and finally comment on a trade-off between the quality and the running time. The last column of Table 3.1 gives results of Concorde and actual path length in *opt* column. On the other side of the table is a greedy approach, which quality (column *quality* computed using Equation 3.2) is mostly in the range between 10% and 20%. However, application of a simple 2-opt heuristic on a randomly generated tour improves the quality to approximately 10% or even better. On the other hand use of GA further improves the quality to approximately 5%. Note, that runs in cases with 70 or more sites terminated due to time limit and hence minimum was not reached. All these results were expected.

The long running time of GA was a reason to graft (or hybridize) the GA with local optimization. The result was substantial decrease in running time. In all cases for GGAemx and for GGdpx the runs were terminated upon reaching the minimum. The reached minimum was, however, the local one. Nonetheless, we showed, that the combination of two methods improved the quality of results in a synergy. The quality of result was bellow 1% off the optimum.

The sixth column in Table 3.1 describes results obtained by GGAemx. In 17 out of 20 considered cases an optimal solution was found. Remaining three instances differ from optimal solution in 0.01, 0.10 and 0.22 percent. The solutions were found in relatively few generations and very fast. Execution times were 0.6 to 15.2 seconds.

The seventh column in Table 3.1 corresponds to results of GGAdpc. In 11 out of 20 considered cases an optimal solution was found. In remained 9 cases, delivered solutions differ from optimal in range from 0.13 to 0.32 percent. The running time and number of generations of GGAdpx, in comparison with GGAemx, are slightly lesser, particularly in the lowermost part of the table which represents more complex instances.

Quantitative results on test cases from *TSPLIB* show that grafted algorithms,

Table 3.1: Five techniques for solving Euclidean TSP: greedy, 2-opt heuristic, GA with edge map crossover, GA with distance preserving crossover, grafted versions of the later and Concorde.

Name	Greedy		2-opt		GAemx		GAdpx		GGAemx		GGAdpX		Concorde	
	quality	time	quality	time	quality	time	quality	time	qual.	gen.	qual.	gen.	opt	time
<i>burma14</i>	8.32%	3.4	5.71%	3.4	0%	81	0%	3.5	0%	7	0%	6	3323	0.1
<i>ulysses16</i>	10.42%	4.1	7.15%	4.1	0%	125	0%	4.4	0%	9	0%	9	6859	0.2
<i>ulysses22</i>	12.54%	14.7	7.87%	14.7	0%	1328	0%	16.4	0%	8	0%	8	7013	0.2
<i>bayg29</i>	13.37%	19.4	6.38%	19.4	0%	1137	0%	17.6	0%	13	0%	14	1610	0.3
<i>bays29</i>	12.87%	29.2	5.37%	29.2	0%	2643	0%	34.1	0%	12	0%	12	2020	0.3
<i>dantzig42</i>	14.06%	79.8	7.11%	79.8	0%	4232	0%	74.6	0%	10	0%	9	699	0.5
<i>att48</i>	13.98%	85.2	8.47%	85.2	0%	5213	0%	91.3	0%	22	0%	23	33522	0.6
<i>eil51</i>	15.24%	100.0+	7.67%	100.0+	5.23%	5489	5.23%	100.0+	0%	33	0%	30	426	0.3
<i>berlin52</i>	14.82%	33.7	7.45%	33.7	4.92%	5021	4.92%	100.0+	0%	15	0%	15	7542	0.4
<i>st70</i>	13.17%	100.0+	7.84%	100.0+	5.12%	5259	5.72%	100.0+	0%	20	0%	19	675	0.5
<i>eil76</i>	14.47%	100.0+	8.15%	100.0+	6.56%	5347	7.24%	100.0+	0%	53	0.19%	49	538	1.3
<i>pr76</i>	13.96%	100.0+	9.95%	100.0+	4.18%	5218	5.36%	100.0+	0%	42	0%	43	108159	1.2
<i>gr96</i>	16.32%	100.0+	7.14%	100.0+	4.98%	5191	5.71%	100.0+	0%	73	0.13%	73	55209	1.6
<i>rat99</i>	14.79%	100.0+	7.41%	100.0+	5.31%	5114	7.12%	100.0+	0%	74	0.17%	70	1211	1.7
<i>kroA100</i>	12.37%	100.0+	8.07%	100.0+	5.12%	5072	6.58%	100.0+	0%	24	0.18%	22	21282	1.7
<i>kroB100</i>	16.58%	100.0+	7.19%	100.0+	6.14%	5041	5.92%	100.0+	0%	39	0.21%	36	22141	1.7
<i>kroC100</i>	10.47%	100.0+	11.19%	100.0+	4.87%	5121	6.78%	100.0+	0.10%	34	0.19%	28	20749	1.8
<i>kroD100</i>	14.81%	100.0+	7.74%	100.0+	5.07%	4976	8.12%	100.0+	0%	31	0.29%	25	21294	1.5
<i>lin105</i>	16.60%	100.0+	9.85%	100.0+	6.72%	4756	6.51%	100.0+	0.01%	26	0.17%	25	14379	1.3
<i>ch150</i>	19.62%	100.0+	11.72%	100.0+	7.22%	4512	8.77%	100.0+	0.22%	88	0.32%	86	6528	7

GGAemx and GGAdpx, have advantages. Even when their's components have serious drawbacks, their grafted combinations exhibits a very good behaviour. Results on examples from *TSPLIB* show that this grafted method combines good qualities from both methods applied and significantly outperforms each individual method.

The running times in Table 3.1 are given for all algorithms but greedy and simple 2-opt heuristics. The later ones had running time in the range between half a second and a second and a half. However, since all algorithms but Concorde were programmed in Java, their running times are not directly comparable. Nonetheless, the relative increase in time as function of a problem size can be compared, and this shows us approximately 25 times increase for GGAemx, 30 times increase for GGAdpx and even 70 times increase for Concorde. Note also, that GGAemx and GGAdpx performed approximately the same, which shows that the recombination operator has no major influence on a final result.

In the second experiment we studied the influence of grafting (hybridization) on running time and quality of solution. In this experiment we used only grafted GA with edge map crossover (GGAemx) and only eleven cases from a TSPLIB (cf. Table 3.2). In the experiment we were increasing the number of generations in which we applied hybridization by 10 percents: from 0% – column *GAemx* in Table 3.2 till 100% – column *GGAemx*. Moreover, we also varied the generations in which we applied the hybridization: either in random generations (column *rnd*), in the beginning ones (*begin*) or in the ending ones (*end*). The column *f.a* gives the running time, while the numbers in columns *q* give the quality computed using Equation 3.2.

We first observe, that application of grafting in the last generations gives the best results. This is reasonable, as in general in this phase of meta-heuristics we apply mostly intensification and not that much more diversification. On the other hand it is interesting that the worse results were obtained when grafting was applied in random generations. Nonetheless, since in practice the algorithm does not know which are the last generations, we would need to simulate the behaviour. There are two possibilities how to do it: either, when time limit is reached run the algorithm for some more runs and apply hybridization or apply hybridization more and more frequently as the number of generations increases. The later approach is also in line with other meta-heuristics like simulated annealing.

The running time obviously linearly increases as we increase the amount of hybridization (see Figure 3.1). Similarly the quality of solution also improves as we increase the hybridization. In Figure 3.2 we see that for the case *pr439* with 439 sites the quality of solution improved from over 10% at no hybridization to approximately 3% at half hybridization and to 1.3% off the optimal at total hybridization. Similar behaviour can be observed at other cases (cf. Figure 3.3). Note, that even small hybridization of 10% drastically improves solution – e.g. for the *pr439* case to only about 4% off the optimal. On the other hand, further hybridization keeps improving result and it is up to the user to decide how much hybridization does she want to employ.

Probably the decision on the amount of hybridization should be made considering the running time. As seen in Figure 3.1 the number of sites increases the steepness of the function. Therefore high hybridization at large cases would probably increase the running time substantially. However, from our experiments seems to follow that

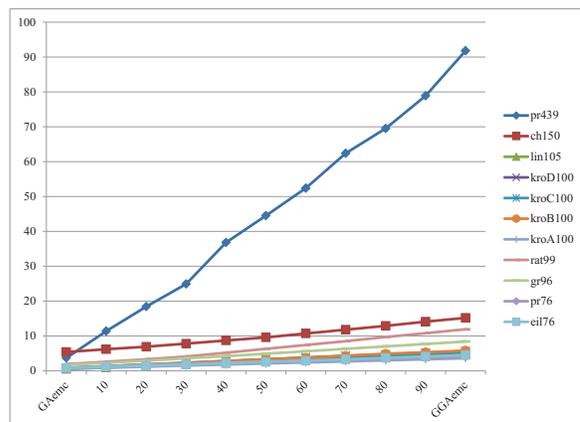


Figure 3.1: The running time as a function of amount of hybridization.

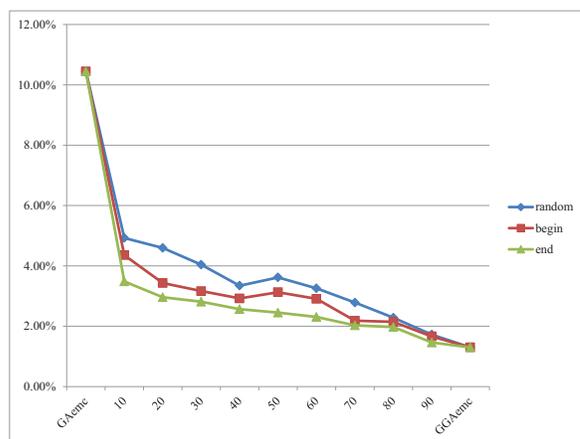


Figure 3.2: Quality results for case *pr439* as the amount of hybridization increases.

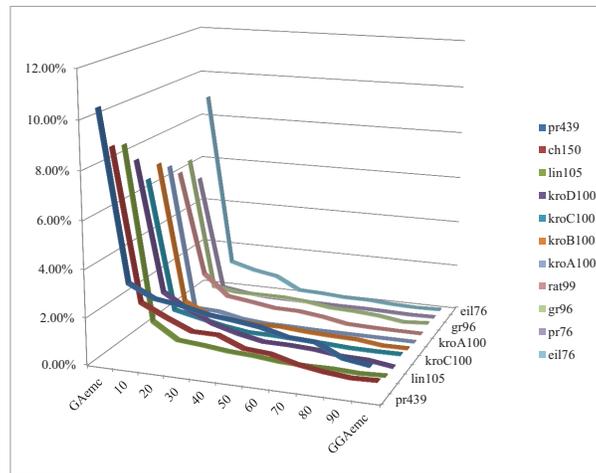


Figure 3.3: Quality results for all cases using end-hybridization.

also lower amounts of hybridization give satisfactory results (see Figure 3.3).

3.5 Conclusions

The goal of this chapter was to investigate influence of grafting a 2-opt based local searcher into the canonical genetic algorithm, for solving the Traveling Salesman Problem. It is known that genetic algorithms are very successful when implemented for many NP-hard problems. However, they are much more effective if some specific knowledge about particular problem is utilized. In our first experiment we compared two direct techniques, with our grafted genetic algorithms. Solutions from Concorde and greedy algorithm were added for better comparison. Quantitative results on test cases from *TSPLIB* show that grafted algorithms have advantages. Even when both components have serious drawbacks, their grafted combinations exhibits a very good behaviour. Results on examples from *TSPLIB* show that this method combines good qualities from both methods applied and significantly outperforms each individual method.

Our experiments further show that the best results are obtained when hybridization occurs in the last generations of the GA. This seems to be in line with classical meta-heuristic algorithms like simulated annealing, which stop their diversification in the last iterations. We showed, that even a small hybridization substantially improves the quality of the result. Moreover, the hybridization in fact does not deteriorate the running time too much.

This chapter opens a number of interesting questions. The first one is related to the size of the problem. Namely, how do our GGA behave in cases, where Concorde can no more compute the optimal solution. Next, can we use some other local optimization technique instead of 2-opt and how would then behave our GGA. A

very interesting question would also be how would our technique work in other NP-hard problems. For example in 3-SAT or CLIQUE.

Chapter 4

Traveling Visitor Problem

Results of this chapter are published in the following articles:

- M. Djordjevic, M. Grgurovic, and A. Brodnik, The Traveling Visitor Problem and Algorithms for Solving It, *Book of Abstracts of 3rd Student Conference on Operational Research, SCOR 2012.*, Nottingham, (2012), 26.
- M. Djordjevic, M. Grgurovic and A. Brodnik, The Traveling Visitor Problem and the Koper Algorithm for Solving It, *Book of Abstracts of 25th Conference of European Chapter on Combinatorial Optimization, ECCO 2012.*, Antalya, (2012), 10.
- M. Djordjevic, J. Zibert, M. Grgurovic, and A. Brodnik, Methods for Solving the Traveling Visitor Problem, *Proceedings of the 1st International Internet and Business Conference, IBC 2012.*, Rovinj, (2012), 174–179.

4.1 Introduction

In the Traveling Salesman Problem (TSP) a set $\{C_1, C_2, \dots, C_N\}$ of cities is considered and for each pair (C_i, C_j) where $i \neq j$, a distance $d(C_i, C_j)$ is given. The goal is to find a permutation π of the cities that minimizes the quantity

$$\sum_{i=1}^{N-1} d(C_{\pi(i)}, C_{\pi(i+1)}) + d(C_{\pi(N)}, C_{\pi(1)}). \quad (4.1)$$

This quantity is referred to as the tour length since it is the length of the tour a salesman would have to travel when visiting the cities in the order specified by the permutation π , returning at the end to the initial city. We will concentrate in this chapter on the symmetric TSP (STSP) in which the distances satisfy $d(C_i, C_j) = d(C_j, C_i)$ for $1 \leq i, j \leq N$. The TSP is known to be *NP-hard* [76]. The case with symmetric distances has been well studied and there are many algorithms which perform well even on large cases [3, 5]. In the literature the Traveling Salesman Problem is usually represented and considered as a graph theoretical problem, see [63, 75]. For more details about TSP see Section 2.1.

An instance of the STSP can be seen as a complete graph $G = (V, E)$ where the set of vertices V is given by the cities and edges between each city in the graph with

corresponding edge weights $d(C_i, C_j)$. The STSP then translates to the problem of finding a Hamiltonian Tour of minimal length in the graph G .

Applications of the TSP and its variations go way beyond the route planning problem of a traveling salesman and span over several areas of knowledge including mathematics, computer science, operations research, genetics, engineering, and electronics. In addition, there are many different variations of TSP which are described and explored in the literature and also variations derived from everyday life. Some of them are: *machine scheduling problems* [8, 63], *the time dependent TSP* [59], the *delivery man problem* which is also known as the *minimum latency problem* and the *traveling repairman problem*, for details on these problems, we refer to Section 2.1.3.

The Traveling Tourist Problem [95] is a problem in which a tourist wishes to see all monuments (nodes) in a city, and so must visit each monument or a neighbour thereof. It is assumed that a monument is visible from any of its neighbours and therefore, the edges represent lines of sight. The resulting walk will therefore visit a subset of all nodes in the graph. The Traveling Tourist Problem shares a similar name with our problem however it is a very different problem.

The STSP can be also solved using the Grafted Genetic Algorithms (GGA) as was shown in [35], [38], [36], [37] and in Section 3. The currently most efficient implementation of the branch-and-cut method which was introduced by Padberg and Rinaldi [116] for solving the symmetric case of TSP is named *Concorde* [4]. Concorde's TSP solver has been used to obtain the optimal solutions to the full set of 110 TSPLIB instances, the largest having 85,900 cities. For more details see Section 2.2.5. Finally, in a graph G besides finding the shortest closed walk we can also find the shortest path between any pair of vertices. This problem is in the literature known as *all-pairs shortest path problem* (APSP) [28]. It aims to compute the shortest path from each vertex u to every other vertex v . The Floyd-Warshall algorithm [28] is an efficient algorithm to find all-pairs shortest paths on a graph G . The all-pairs shortest path problem and the Floyd-Warshall algorithm are in more details described in Sections 2.1.5 and 2.1.5.

4.2 Traveling Visitor Problem

Suppose that visitors have arrived in a hotel in some town, with a desire to visit all interesting sites in a city exactly once and to come back to the hotel at the end of their journey. Visitors in general move from one place to another using the existing streets, walking trails and pedestrian zones. The goal is to minimize the visitors traveling distance.

The Traveling Visitor Problem is a version of the Traveling Salesman Problem with a difference that the traveling visitor, during its visit of sites, can not fly over the buildings in the city, instead visitors must go around these obstacles. This difference is demonstrate in Figure 4.1. This means that the Euclidean distances [57, 113], as we know them in the Euclidean TSP, are in this case not valid (direct edge from i to j in Figure 4.1). Visitors use the walking paths and pedestrian zones of variable length. These limits determine the weight of edges connecting the vertices in the graph.

The Traveling Visitor Problem is stated as: given a connected, weighted graph $G = (V, E, c)$, with a set of vertices $V = S \cup X$ and $S \cap X = \emptyset$, S is the set of interesting sites in the city (vertices i and j in Figure 1.1), X is the set including the nodes corresponding to crossroads in the city (vertices k and m in Figure 1.1), a set of edges E , and a cost of traveling c . The goal is to find the shortest closed walk through all vertices from S , according to c in graph G , although we may travel through vertices from X .

The concepts we summarised above can be modified easily to take the directions of the edges into account. The asymmetric traveling visitor problem (ATVP) is then similar to the symmetric TVP above, i.e. it is the problem of finding a closed walk of minimal length in a weighted graph. The Euclidean TVP, or planar TVP, is the TVP with the distance being the ordinary Euclidean distance. The Euclidean TVP is then a particular case of the metric TVP, since distances in a plane obey the Euclidean triangle inequality.

This problem, by the knowledge of the authors, has no references in publications due date of writing it.

4.2.1 Algorithms for solving TVP

One simple approach of solving TVP is to visit all places as are ordered in the city's tourists maps and then come back to the starting site. The result of this method depend directly on the order in which the interesting sites are listed on the map and does not necessary find the shortest closed walk through all tourist sites.

The first proposed method for solving the Traveling Visitor Problem is the Naïve algorithm, shown in Algorithm 10. In the first line of pseudocode we can distinguish next parameters: S is the set of interesting sites in the city, X is the set of crossroads in the city, a set of edges E , and W represents the distance matrix of the graph G , $(S \cup X \times S \cup X)$. The First step of the algorithm is to solve a TSP considering every node in S . Then a tour, denoted by T , is obtained. In the next step, a distance matrix Z is constructed by solving the APSP for every node in S . Note that to calculate these distances we do use distance matrix W (nodes in $S \cup X$). Finally, we calculate the final cost combining the shortest path solution obtained in the second step with the optimal tour obtained in the first step.

Algorithm 10 Naïve Algorithm

```
1: procedure NAÏVE( $S, X, E, W$ )
2:    $T \leftarrow TSP(S)$ 
3:    $Z \leftarrow APSP(S \cup X, E, W)$ 
4:    $cost \leftarrow 0$ 
5:   for all  $(i, j) \in T$  : do
6:      $cost \leftarrow cost + Z_{ij}$ 
7:   end for
8: end procedure
```

The second proposed method for solving the Traveling Visitor Problem is our Koper algorithm, shown in Algorithm 11. The first line of pseudocode contains the

same parameters as Naïve algorithm. In the first step we find all-pairs shortest paths in our graph G . As an input a distance matrix W is used and as the output a distance matrix Z is obtained ($Z \leftarrow S \times S$). In the next step we solve the Traveling Salesman Problem on the distance matrix Z . Furthermore, we get the solution T , which is a solution for Traveling Visitor Problem. Algorithms presented in this section was named so by the author of dissertation.

Algorithm 11 Koper Algorithm

```

1: procedure KOPER( $S, X, E, W$ )
2:    $Z \leftarrow APSP(S \cup X, E, W)$ 
3:    $T \leftarrow TSP(Z)$ 
4: end procedure
    
```

4.2.2 Adapted Floyd-Warshall algorithm

In Section 4.1 we met a classical all-pairs shortest path problem in graph $G(V, E)$, which can be solved using Floyd-Warshall algorithm (cf. Algorithm 12, where W is the distance matrix of the graph G) in time $|V|^3$. However, our problem is

Algorithm 12 Floyd-Warshall

```

1: procedure FLOYD-WARSHALL( $V, W$ )
2:   for all  $k \in V$  do
3:     for all  $i \in V$  do
4:       for all  $j \in V$  do
5:          $W_{ij} := \min(W_{ij}, W_{ik} + W_{kj})$ 
6:       end for
7:     end for
8:   end for
9: end procedure
    
```

somewhat more restrictive as V consists of disjoint subsets X and S and we are only interested in the shortest paths between vertices of S while a path itself can visit an arbitrary number of vertices in X . This restriction permits us to modify slightly the Floyd-Warshall algorithm and obtain the following Lemma.

Lemma 4.2.1 *Let $G(X \cup S, E)$ be a graph where $X \cap S = \emptyset$, $x = |X|$ and $s = |S|$. Let a S -path be a sequence of edges from E that starts and ends in S but can also visit vertices in X . Then there exists an algorithm that computes all-pairs shortest S -path between any pair of $u, v \in S$ in time $s^3 + x^3 + s^2x + x^2s$. Considering also the leading term, this can be up to twice as fast as Floyd-Warshall algorithm.*

PROOF. We split the graph G into several subgraph components consisting of vertices either from S or X , and edges between these components. Figure 4.2.2

shows an example of splitting S and X into components S' and S'' , and components X' and X'' respectively ($S = S' \cup S''$, $X = X' \cup X''$, where $S' \cap S'' = \emptyset$, $X' \cap X'' = \emptyset$). To illustrate the proof we use the example from the figure. Although

in general S and X can break in several components, it will be easy to see that the number of components does not influence neither correctness nor the time complexity of our algorithm.

Consider the path π_{uv} between nodes $u, v \in S$. It starts and ends in S , but it can pass through X (cf. Figure 4.2.2). Moreover, it can oscilate between X and S . Assume that π_{uv} for the first time re-enters S at vertex $w \in S$. Due to the minimization principle, paths π_{uw} and π_{wv} are also the shortest paths between u and w , and w and v respectively. Therefore, if we compute these paths first, the relaxation principle in Floyd-Warshall algorithm will pick up the correct path for π_{uv} .

Furthermore, let $\pi_{uw} = u \rightarrow \pi_{ab} \rightarrow w$, that is path π_{uw} enters X at a and exits at b . As before π_{ab} is also the shortest path between $a, b \in X$, which is completely in X . Similarly paths $u \rightarrow \pi_{ab}$ and $\pi_{ab} \rightarrow w$ are also the shortest paths between the respective vertices. Therefore, if we have computed the last two paths, the relaxation procedure will pick the correct path when computing the shortest path π_{uw} .

This brings us to the algorithm in Algorithm 13. We first compute all-pairs

Algorithm 13 Adapted Floyd-Warshall Algorithm

```

1: procedure ADAPTED( $S, X, W$ )
2:   FLOYD-WARSHALL( $X, W$ )
3:   for all  $k \in X$  do
4:     for all  $i \in X$  do
5:       for all  $j \in S$  do
6:          $W_{ij} := \min(W_{ij}, W_{ik} + W_{kj})$ 
7:       end for
8:     end for
9:   end for
10:  for all  $k \in X$  do
11:    for all  $i \in S$  do
12:      for all  $j \in S$  do
13:         $W_{ij} := \min(W_{ij}, W_{ik} + W_{kj})$ 
14:      end for
15:    end for
16:  end for
17:  FLOYD-WARSHALL( $S, W$ )
18: end procedure
    
```

shortest paths (cf. π_{ab}) between all vertices in X , which takes x^3 time. Next (lines 3-9), we compute the shortest paths between all nodes $a \in X$ and $w \in S$ (cf. $\pi_{ab} \rightarrow w$), which takes another x^2s time. In lines 10-16 we compute the shortest paths between all nodes $u \in S$ and $b \in X$ (cf. $u \rightarrow \pi_{ab}$), which takes another xs^2 time. The final call to Floyd-Warshall algorithm on S in time s^3 computes the final result as all possible sub-paths are already computed. The total time of the our algorithm is $x^3 + x^2s + xs^2 + s^3$ which has extreme $4s^3$ at $x = s$. On the other hand the usual Floyd-Warshall algorithm takes in this case $(x + s)^3 = 8s^3$ ■

4.3 Experiments

For testing our strategies, described in Section 4.2.1 we used the real instances of the Traveling Visitor Problem, which were made from official tourist maps of cities of Koper, Belgrade and Venice. In the Belgrade example two different cases were considered and they differed in the size of the problem, i.e. the number of vertices in the graph. From the publicly available library, TSPLIB, of sample benchmarks for the TSP and related problems, two instances of the symmetric traveling salesman problem were selected, modified and tested.

These two instances were modified in such a way that a new graph G' was made satisfying the conditions of connected, weighted graph. Furthermore we split V into a set of vertices S and set of vertices X , such that $|S| = |X| = |V|/2$. A vertex degree 5 is arbitrarily assigned, inspired by the case of real instances, and means that from every vertex from V there is exactly 5 edges going to the other vertex from V . The 5 edges per vertex were chosen randomly, according to a uniform probability distribution.

Altogether 5 instances were tried out, with different sizes which range from 120 to 1002 vertices per instance. We compared two methods for solving the Traveling Visitor Problem. The first method is the Naïve algorithm, shown in Algorithm 10. The second tested method is the Koper algorithm, shown in Algorithm 11. For solving the TSP, as one step in both algorithms, we used the Concorde Algorithm, presented in Section 2.2.5. Furthermore, for solving the APSP, as a part of both algorithms, we used the Adapted Floyd–Warshall algorithm, which was presented in Section 4.2.2.

4.4 Evaluation

The results of the experiment are summarized in Table 4.1. Five instances were tried out, with different sizes, ranging from 120 to 1002 vertices per instance. The names of these instances are in the first column. The second column contains the size of the problem, i.e. the number of vertices. The third column in Table 4.1 corresponds to the number of vertices in set S and in the top three instances the number of interesting sites from tourist maps. The fourth column contains names of the two tested methods. The fifth column corresponds to the length of the tour i.e., the cost of a solution, which was obtained in the experiment. In all six cases, Koper Algorithm obtains the shortest tours. The last column corresponds to the quality of the result. The comparison of both algorithms is computed by the relative difference of cost solutions that is defined as

$$q_A = \frac{l_A - l_C}{l_C} \quad (4.2)$$

where l_C is a length of the tour obtained by Koper algorithm and l_A is a length of the tour obtained by Naïve algorithm. The relative difference is expressed always in percents. For example, 17.22% difference means that Naïve algorithm performed 17.22% worse than Koper algorithm. The first tested method, the Naïve algorithm, performed poorly in comparison to the Koper algorithm. The quality differs from

Name	(V)	(S)	Methods	Tour Cost	Difference
<i>Koper</i>	120	55	Naïve	4738	17.22%
			Koper	4042	
<i>Belgrade</i>	163	53	Naïve	100389	6.52%
			Koper	94246	
	250	90	Naïve	122119	8.77%
			Koper	112275	
<i>Venice</i>	210	72	Naïve	26648	24.24%
			Koper	21448	
<i>lin318</i>	318	159	Naïve	921499	249.08%
			Koper	263983	
<i>pr1002</i>	1002	501	Naïve	11818732	354.46%
			Koper	2600585	

Table 4.1: Two techniques for solving the Traveling Visitor Problem

6.52% in the case of Belgrade163 to 354.46% in the case of pr1002 instance. The difference in the quality of the solutions of two tested algorithms grows with instances that are more complex.

Although these algorithms are similar in terms of components (both rely on solving an APSP and TSP), the difference on the quality of the solutions indicates that there is a gain by using the Koper algorithm. Note also that the running time for both algorithms is the same.

4.5 Conclusions

The goal of this chapter is to describe a new problem from graph theory, named the Traveling Visitor Problem. Although the new problem is similar to the Traveling Salesman Problem, when we try to solve it with the Naïve algorithm we get solutions far from optimal. The minimum cost solutions for the Traveling Visitor Problem instances tested in the chapter are provided by our Koper algorithm. The tested benchmarks are obtained from three real instances coming from tourist maps of cities of Koper, Belgrade and Venice and two modified instances from TSPLIB. In all tested cases the Koper algorithm significantly outperforms the Naïve algorithm for solving the Traveling Visitor Problem - quality of solutions differs from 6.52% to 354.46%.

The chapter opens a number of interesting questions for future research. The first one is related to the size of the problem. Namely, how will some heuristic algorithm behave in cases, where Concorde can no longer compute the optimal solution? A very interesting question is also how to apply our technique to Asymmetric Traveling Visitor Problem.

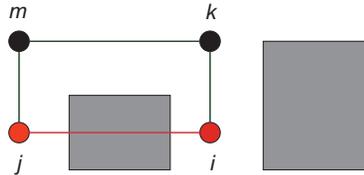


Figure 4.1: TSP and TVP, Two rectangles represent buildings (obstacles) in the city. Red nodes represent interesting sites in the city (vertices from set S), black nodes represent crossroads in the city (vertices from set X), the red line represent the Euclidean shortest connection between two interesting sites (this is the case in TSP), black lines represent the connection between two interesting sites, going through two crossroads (this is the case in TVP)

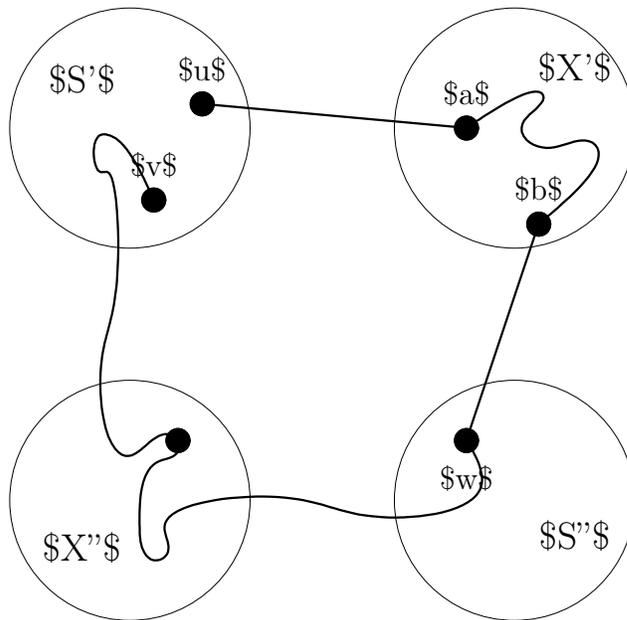


Figure 4.2: Each node represents connected component consisting of nodes/vertices only from either X or S , while edges represent (arbitrarily many) connections between components.

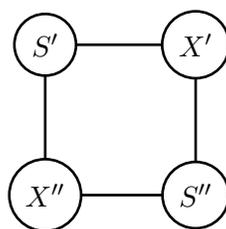


Figure 4.3: Each node in the graph represents an arbitrary amount of vertices from a single set that are arbitrarily interconnected. The edges represent (arbitrarily many) connections to other such sets.

Chapter 5

Conclusion

The first goal of the Thesis was to investigate influence of grafting a 2-opt based local searcher into the standard genetic algorithm, for solving the Traveling Salesman Problem. It is known that genetic algorithms are very successful when implemented for many NP-hard problems. However, they are much more effective if some specific knowledge about particular problem is utilized. The TSP is well researched problem with many such improvements, especially when the restricted version of the problem with Euclidean distance is considered. In our experiment in Chapter 3 we compared two direct techniques, with our grafted genetic algorithms. Solutions from Concorde and greedy algorithm were added for better comparison. Quantitative results on test cases from *TSPLIB* show that grafted algorithms have advantages. Even when both components have serious drawbacks, their grafted combinations exhibit very good behaviour. Results on examples from *TSPLIB* show that this method combines good qualities from both methods applied and significantly outperforms each individual method. Our experiments further show that the best results are obtained when hybridization occurs in the last generations of the GA. This seems to be in line with classical meta-heuristic algorithms like simulated annealing, which stop their diversification in the last iterations. On one hand the less frequent application of hybridization decreased the average running time of the algorithm from 14.62 sec to 2.78 sec at 100% and 10% hybridization respectively, while on the other hand the quality of solution on average deteriorated only from 0.21% till 1.40% worse than the optimal solution. We showed, that even a small hybridization substantially improves the quality of the result. Moreover, the hybridization in fact does not deteriorate the running time too much.

The second goal of the Thesis was to describe a new problem from graph theory, named the Traveling Visitor Problem. Although the new problem is similar to the Traveling Salesman Problem, when we try to solve it with the Naïve algorithm we get solutions far from optimal. The minimum cost solutions for the Traveling Visitor Problem instances tested in the paper are provided by our Koper Algorithm. The tested benchmarks are combined from three real instances made using tourist maps of cities of Koper, Belgrade and Venice and two instances of modified cases from *TSPLIB*. In all tested cases the Koper Algorithm significantly outperforms the Naïve Algorithm for solving the Traveling Visitor Problem - quality of solutions differs from

6.52% to 354.46%.

5.1 Hypotheses and Original Contribution to the Science

In chapter 1 two hypotheses were introduced:

- Hypothesis 1: The method for solving of TSP that is made of two independent methods, genetic algorithm and 2-opt heuristic, outperforms each of the combined methods in terms of the quality of solution.
- Hypothesis 2: The quality of solution of a special method to the problem of a traveling visitor problem, outperforms algorithms for solving general TSP problem when they are used for solving traveling visitor problem.

Using a specific methodology, detailedly described in Chapter 3, we prove the correctness of the hypotheses 1. Furthermore, in Chapter 4 we prove the correctness of the hypotheses 2. Research objective of the thesis is the proof of the hypotheses 1 and 2.

Contributions to the science consist of the following results:

- Construction of the grafted genetic algorithm for solving the traveling salesman problem.
- Verification that the traveling salesman problem can be successfully solved using the grafted genetic algorithm.
- Construction of the special method for solving the traveling visitor problem.
- Construction of the real case instances of the traveling visitor problem for cities of Koper, Belgrade and Venice.
- Verification that all instances of the traveling visitor problem, which are solved using the special method, represent a very satisfactory solution.

The results of doctoral dissertation represents the contribution to bridging the gap between theoretical computer science and its application in practice. Also to better understanding and modeling of real problems in the economy, represented as the NP-hard problems from graph theory as well as a contribution to the optimization methods for solving these hard problems.

The results of this PhD Thesis are published in the following articles:

- M. Djordjevic, Influence of Grafting a Hybrid Searcher Into the Evolutionary Algorithm, *Proceedings of the 17th International Electrotechnical and Computer Science Conference, Portoroz, Slovenia* (2008), 115–118.
- M. Djordjevic, and M. Tuba, and B. Djordjevic, Impact of Grafting a 2-opt Algorithm Based Local Searcher Into the Genetic Algorithm, *Proceedings of the 9th WSEAS international conference on Applied informatics and communications, AIC 2009, Moscow, Russia* (2009), 485–490.

- M. Djordjevic, and A. Brodnik, Quantitative Analysis of Separate and Combined Performance of Local Searcher and Genetic Algorithm, *Book of Abstract of International Conference on Operations Research, OR 2011, Zurich, Switzerland* (2011), 130.
- M. Djordjevic, and A. Brodnik, Quantitative Analysis of Separate and Combined Performance of Local Searcher and Genetic Algorithm, *Proceedings of the 33rd International Conference on Information Technology Interfaces, ITI 2011, Dubrovnik, Croatia* (2011), 515–520.
- M. Djordjevic, A. Brodnik and M. Grgurovic, The Traveling Visitor Problem and the Koper Algorithm for Solving It, *Book of Abstracts of 25th Conference of European Chapter on Combinatorial Optimization, ECCO 2012, Antalya, Turkey* (2012), 10.
- M. Djordjevic, A. Brodnik and M. Grgurovic, The Traveling Visitor Problem and Algorithms for Solving It, *Book of Abstracts of 3rd Student Conference on Operational Research, SCOR 2012, Nottingham, UK* (2012), 26.
- M. Djordjevic, and M. Grgurovic, and A. Brodnik, Performance Analysis of Partial Use of Local Optimization Operator on Genetic Algorithm for Traveling Salesman Problem, *Business Systems Research*, Print ISSN 1847-8344; Online ISSN 1847-9375, in press.

Povzetek v slovenskem jeziku

5.2 Uvod

Ko sem prišel leta 2007 na podiplomski doktorski študij računalništva, iz Beograda v Koper, sem prišel z veliko željo, da spoznam mesto, v katerem bom preživel naslednja štiri leta. Mestno jedro mi je bilo takoj všeč predvsem zaradi številnih znamenitosti, natančno 55, ki se nahajajo tudi na turističnem zemljevidu Kopra. Ker je doktorski študij naporen in nisem imel veliko prostega časa, sem se začel spraševati kako bi bilo, če bi lahko svoj obhod po mestnih znamenitostih optimiziral na tak način, da porabim najmanj možnih korakov in tako prihranim nekaj časa. Problem sem poimenoval *problem potujočega obiskovalca* (angl. *Traveling Visitor Problem, (TVP)*).

V zgodnjih 30. letih 20. stoletja, je avstrijski matematik Karl Menger izzval takratno raziskovalno skupnost, naj z matematičnega vidika preuči, sledeč problem [100]: Glasnik želi obiskati vsako mesto s seznama na katerem je n mest natanko enkrat in se nato vrniti v svoje mesto pri tem so cene potovanj iz mesta i v mesto j znane vnaprej. Vprašanje je torej kateri obhod je najcenejši? Problem Trgovskega Potnika (angl. *Traveling Salesman Problem, (TSP)*) je formalno definiran na polnemu grafu $G = (V, E)$, kjer je $V = \{v_1, v_2, \dots, v_n\}$ množica vozlišč, E množica povezav in s cenilno funkcijo $c(i, j)$, ki povezavi $(i, j) \in E$ priredi določeno ceno.

TSP lahko obravnavamo tudi kot problem permutacij. Naj bo P_n množica vseh permutacij iz množice $\{1, 2, \dots, n\}$. Potem je problem trgovskega potnika poiskati $\pi = (\pi(1), \pi(2), \dots, \pi(n))$ v P_n , za katero velja, da je $c_{\pi(n)\pi(1)} + \sum_{i=1}^{n-1} c_{\pi(i)\pi(i+1)}$, minimalen.

TSP je eden izmed najpomembnejših predstavnikov večje množice problemov, imenovane kombinatorični optimizacijski problemi [67]. Ker spada TSP v razred NP-težkih (angl. *NP-Hard*) problemov [76], učinkovitega algoritma za TSP ne poznamo. Natančneje, takšen algoritem obstaja če in samo če sta razreda P in NP enaka. S praktičnega vidika to pomeni, da ne poznamo natančnega algoritma za katerikoli TSP primer z n vozlišči, ki se obnaša značilno bolje, kot algoritem, ki izračuna vseh $(n - 1)!$ možnih obhodov ter vrne obhod z najmanjšo ceno.

V praksi lahko za reševanje tega problema uporabimo tudi drugačen pristop. Določeni TSP primer, z n vozlišči ima lahko katerikoli obhod, ki poteka skozi vsa vozlišča n in predstavlja možno rešitev, ki je zgornja meja (angl. *upper bound*) za

najnižjo možno ceno. Algoritem, ki v polinomskem času (angl. *polynomial time*) konstruira možne rešitve s to zgornjo mejo se imenuje heuristika [12,121]. Načeloma, ti algoritmi tvorijo rešitve, vendar brez zagotovila o kakovosti rešitve glede na razliko med njihovo ceno in optimalno ceno.

Poznamo dve vrsti TSP: simetrični TSP in asimetrični TSP. V simetrični obliki, znani pod imenom STSP [73, 74, 93, 97], je razdalja med vozliščema i in j enaka razdalji med vozliščema j in i . V primeru asimetričnega TSP (ATSP) [14, 16, 18, 24], takšna simetrija ne obstaja. Poleg tega obstaja še vrsta različnih variacij TSP, ki so opisane in raziskane v literaturi ter predstavljene v doktorski disertaciji v Poglavju 2. Spodaj povzemam nekatere izmed njih.

Grupirani TSP (angl. *Clustered TSP*) [65], ozko-grlni TSP (angl. *Bottleneck TSP*) [63], posplošen TSP (angl. *Generalized TSP*) [62, 79], problem glasnika (angl. *Messenger Problem*) [100], ki je znan tudi kot problem izgubljenega prodajalca (angl. *Wondering Salesman Problem*) [78], problem zamenjave (angl. *The swapping problem*) [2], problem minimalne latentnosti (angl. *Minimum Latency Problem*) [11] znan tudi kot problem dostavljalca (angl. *Delivery Man Problem*) [63] ali problem potujočega mojstra (angl. *Traveling Repairman Problem*) [53], problem seizmičnih plovil (angl. *Seismic Vessel Problem*) [61], ki je posplošitev problema skladiščnega dvigala (angl. *Stacker Crane Problem*) [25], problem potujočega turnirja (angl. *Traveling Tournament Problem*) [42], problem lokacije objekta (angl. *Facility Location Problem*) [41]. In končno problem potujočega obiskovalca, ki je podrobno opisan, raziskovan in rešen v doktorski disertaciji v Poglavju 4, za ta problem ne poznamo nobenega vira v literaturi.

Prvi koraki v reševanju TSP so bili klasični poskusi. Te metode so sestavljene iz natančnih in heurističnih algoritmov. Natančne metode, kot so presek ravnine (angl. *cutting planes*) [32], vejitev in povezovanje (angl. *branch and bound*) [27, 32], lahko optimalno rešijo relativno majhne probleme (v odvisnosti od velikosti n), med tem ko metode, kot so različne variante Lin-Kernighan algoritma [6, 46, 70, 79] in Concorde tehnike [3–5] nam dajo relativno dobre rezultate, tudi za večje probleme. Posamezni algoritmi, zasnovani na požrešnih principih, kot sta najbližji sosed (angl. *nearest neighbour*) [63] in vpeto drevo (angl. *spanning tree*) [69], se prav tako uporabljajo za reševanje TSP.

Natančne metode za reševanje TSP rezultirajo z eksponentnimi računskimi kompleksnostmi, tako da so v izogib obstoječim slabostim potrebne nove metode. Te metode vključujejo različne principe optimizacijskih tehnik, naravno orientirani optimizacijski algoritmi, populacijsko orientirani optimizacijski algoritmi, ter drugi. Različna bitja in naravni sistemi, ki se razvijajo v naravi so zanimivi in dragoceni izvori navdiha, za raziskovanje in ustvarjanje novih sistemov in algoritmov za reševanje TSP, ter njegovih variacij. Nekatere od teh metod so predstavljene v doktorski disertaciji v Poglavju 2. Naj jih naštejemo nekaj.

Evolutivno računanje (angl. *Evolutionary Computation*) [101, 106, 127, 136], genetski algoritmi (angl. *Genetic Algorithms*) [43, 51, 52, 103, 109, 117, 126, 130, 134, 135, 139, 140], memetični algoritmi (angl. *Memetic Algorithms*) [62, 87–89, 104, 113], sistemi mravelj (angl. *Ant Systems*) [39], simulirano ohlajanje (angl. *Simulated Annealing*) [84], in naposled *cepljeni genetski algoritmi* (angl. *Grafted Genetic Algorithms*) [35], [38], [36], [37]. Slednji predstavljajo vrsto hibridnih genetskih algo-

ritmov in so raziskani, podrobno opisani ter demonstrirani v doktorski disertaciji v Poglavlju 3.

5.3 Vsebina disertacije

Doktorska disertacija po Pravilniku o pripravi in zagovoru doktorske disertacije na Univerzi na Primorskem, vsebuje naslednja poglavja:

- Zahvala
- Povzetek
- Kazalo vsebine
- Poglavlje 1 - Uvod
- Poglavlje 2 - Ozadje
 - 2.1 Problem Trgovskega Potnika
 - 2.2 Optimizacijski Algoritmi
- Poglavlje 3 - Cepljeni Genetski Algoritmi
- Poglavlje 4 - Problem Potujočega Obiskovalca
- Zaključek
- Literatura
- Kazalo
- Izjava

V poglavju *Ozadje* smo predstavili osnovne pojme trgovskega potnika in optimizacijskih algoritmov, s pomočjo katerih so lahko nadaljnja poglavja disertacije postala razumljiva tudi širšemu krogu bralcev. Preostali poglavji so namenjeni predstavitvi doseženih ciljev disertacije. Poglavlja so razdeljena na več podpoglavij.

V doktorski disertaciji sta obdelani dve temi iz teoretičnega računalništva. Optimizacijsko hevristična metoda imenovana cepljeni genetski algoritmi (GGA) in kombinatorično optimizacijski problem, imenovan problem potujočega obiskovalca (TVP). Cilja doktorske disertacije sta naslednja:

Cepljeni genetski algoritmi: Cilj je pokazati kakovost dobljenih rešitev in hitrost izvajanja cepljenega genetskega algoritma, kadar se uporablja za probleme Simetričnih TSP-jev, ki so na voljo na svetovnem spletu, v obliki splošno priznanih ocenjevalnih primerov (angl. *benchmarks*), kot tudi dejanskih primerov, nastalih za problem potujočega obiskovalca.

Problem potujočega obiskovalca: Cilj je opisati in definirati problem iz realnega življenja, ustvariti realne primere za mesta v okolici in rešiti primere problemov z uporabo nove metode ter znanih metod, ki bodo skupaj prikazane v doktorski disertaciji. Raziskovalni cilj disertacije je dokazovanje spodaj navedenih Hipotez 1 in 2.

Hipoteza 1: Metoda za reševanje TSP, sestavljena iz dveh neodvisnih metod, genetskega algoritma in 2-opt hevristike, združuje kakovosti obeh metod na takšen način, da ju znatno prekaša, glede na kakovost rešitve.

Hipoteza 2: Glede na kakovost rešitve posebna metoda za reševanje problema potujočega obiskovalca, prekaša splošne algoritme za reševanje problema trgovskega potnika, ko jih uporabimo za reševanje problema potujočega obiskovalca.

5.4 Raziskava

5.4.1 Cepljeni Genetski Algoritmi

Botanično cepljenje je postopek, ko je tkivo prve rastline pritrjeno na tkivo druge rastline. Cepljenje lahko zmanjša čas cvetenja in skrajša čas rejskega programa. Lokalni iskalec je razširitev konvencionalnega genetskega algoritma, saj ne obstaja potreba po uporabi komponent genetskega algoritma. To omogoča optimizacijo posameznih genomov, izven evolucijskega procesa. V našem algoritmu po izvedeni rekombinaciji (vrstica 7 v Algoritmu 9), se lokalni iskalec uporablja za optimizacijo vsakega genoma potomcev (vrstica 8 v Algoritmu 9). Zaradi uporabe omenjene zunanje optimizacijske metode, genetski algoritem ni več čist, zato govorimo o cepljenem genetskem algoritmu [35], [38]. Omenjena oblika optimizacije se izvaja lokalno ter spreminja genom s pomočjo hevrističnega spreminjanja rešitve. 2-opt lokalni iskalec 2.2.3 je lokalna optimizacijska metoda za TSP, ki je bila vcepljena v standardni genetski algoritem (vrstica 8 v Algoritmu 9). Ta lokalna optimizacijska metoda, izvaja 2-opt hevristiko, ki izmenjuje povezave grafa z namenom zmanjšanja dolžine obhoda. Postopek izmenjave je sestavljen iz odstranjevanja dveh povezav iz trenutnega obhoda in ponovnega povezovanja na najboljši možen način, pogledj Figuro 2.1.

V opravljenem poskusu smo testirali vpliv cepljenja algoritma lokalnega iskalca z genetskim algoritmom za reševanje problema trgovskega potnika. Za testiranje naše strategije in primerjave le te z ostalimi rešitvami, smo uporabili primere simetričnega problema trgovskega potnika, ki so na voljo na spletu, v knjižnici TSPLIB [122]. Uporabili smo 20 primerov, z različno kompleksnostjo in obsegom od 14 do 150 mest, Tabela 3.1. Primerjali smo našo metodo (Cepljeni Genetski Algoritem), katere predstavnik sta algoritma GGAemc in GGAopc, s štirimi drugimi metodami. Za zgornjo mejo kakovosti rešitve smo uporabili požrešno hevristiko (angl. *Greedy Heuristic* 2.2.2), za spodnjo mejo smo uporabili globalni minimum, pridobljen s Concorde 2.2.5. Nato smo primerjali našo cepljeno metodo z 2-opt in genetskim algoritmom.

Rezultati eksperimenta so predstavljeni v Tabeli 3.1. Šesti stolpec v tabeli predstavlja rešitve našega cepljenega algoritma, ki je bil programiran s pomočjo križanja povezav (angl. *edge map crossover* 2.2.4), kot operator za rekombinacijo (GGAemc). V sedemnajstih od dvajset obravnavanih primerov je bila najdena optimalna rešitev,

preostali trije primeri odstopajo od optimalne rešitve za 0,01, 0,10 in 0,22 odstotka. Rešitve so bile najdene hitro, porabili smo med 0,6 in 15,2 sekund ter v relativno majhnem številu generacij. Sedmi stolpec v Tabeli 3.1, pripada našemu cepljenemu genetskemu algoritmu, ki vsebuje križanje ohranjanja razdalje (angl. *distance preserving crossover* 2.2.4), kot operator za rekombinacijo (GGAdpc). V enajstih od dvajset obravnavanih primerov je bila najdena optimalna rešitev, v preostalih devetih primerih so odstopanja od optimalne rešitve od 0,13 do 0,32 odstotka. V primerjavi z GGAemc, sta čas izvajanja in število generacij GGAdpc nekoliko manjša, posebej v nižjem predelu tabele, ki predstavlja kompleksnejše primere.

Kvantitativni rezultati na testnih primerih iz TSPLIB kažejo, da imata cepljena algoritma, GGAemc in GGAdpc prednosti. Kljub številnim pomankljivostim njunih komponent, se njune kombinacije cepljenja zelo dobro obnesejo. Rezultati primerov iz TSPLIB kažejo, da omenjena metoda cepljenja združuje dobre lastnosti iz obeh uporabljenih metod in občutno prekaša vsako izmed njiju.

5.4.2 Problem Potujočega Obiskovalca

Obiskovalci so prispeli v hotel v nekem novem mestu z željo, da obišejo vse zanimivosti mesta natanko enkrat in se po ogledu vrnejo v hotel. Na ogled mesta se odpravijo peš - po ulicah, sprehajalnih zonah in poteh za pešce. Cilj je skrajšati pot obiskovalca.

Problem potujočega obiskovalca je izpeljan iz *problema trgovskega potnika*, pri čemer velja pravilo, da obiskovalec izbira samo med potmi, ki jih je možno prehoditi. To pomeni, da so evklidske razdalje [57, 113], kot jih poznamo v evklidskem TSP, v našem primeru napačne. Obiskovalci uporabljajo sprehajalne poti in območja za pešce, ki so različno dolge. Te omejitve določajo težo povezav, ki povezujejo vozlišča v grafu.

Definicija TVP je sledeča: Imamo graf $G = (V, E, c)$, kjer je množica vozlišč $V = S \cup X$ in $S \cap X = \emptyset$, kjer sta S zanimivosti mesta in X križišča, E množica povezav ter c cena potovanja. Cilj je poiskati najkrajši zaprt sprehod (angl. *closed walk*, poglavje 2.1.1) skozi vsa vozlišča S (glede na c) v grafu G , pri čemer se lahko sprehodimo skozi X .

Prva predlagana metoda za reševanje problema potujočega obiskovalca je Naivni algoritem (angl. *Naïve Algorithm*), prikazan v Algoritmu 10. V prvi vrstici psevdokode, lahko razberemo naslednje parametre: S pripada zanimivim lokacijam v mestu, X pripada križiščem, E pripada množici povezav, W pa predstavlja matriko povezav grafa G , $(S \cup X) \times (S \cup X)$. V prvem koraku algoritma je problem potujočega obiskovalca rešen kot primer problema trgovskega potnika. V naslednjem koraku, iz matrike povezav W izdelamo matriko povezav Z dimenzij $(S \times S)$, ki predstavlja rešitev problema najkrajših poti vseh parov (angl. *All-Pairs Shortest Paths Problem (APSP)*, poglavje 2.1.5). V zanki (od vrstice 6 do 8) je prikazana rešitev za TVP. Druga predlagana metoda za reševanje problema potujočega obiskovalca je algoritem Koper (angl. *Koper Algorithm*), prikazan v Algoritmu 11. Prva vrstica psevdokode vsebuje enake parametre kot Naivni algoritem. V prvem koraku poiščemo najkrajše poti med vsemi pari vozlišč množice S v grafu G . Vhodno matriko razdalj označimo z

W in izhodno matriko razdalj Z . V naslednjem koraku, rešimo problem trgovskega potnika za matriko razdalj Z . Poleg tega smo dobili T , ki je rešitev za problem potujočega obiskovalca.

Za testiranje naše strategije smo uporabili primere problema potujočega obiskovalca, ki smo jih izdelali iz uradnih turističnih zemljevidov za mesta Koper, Beograd in Benetke. V primeru Beograda smo izdelali dva primera, ki se razlikujeta po velikosti problema, oziroma po številu vozlišč v grafu. Iz javno dostopne knjižnice, TSPLIB, smo si izbrali, spremenili in testirali dva primera problema simetričnega trgovskega potnika. Izvedli smo poskuse za 5 primerov, z različnimi velikostmi, ki so od 120 do 1002 vozlišč za posamezen primer.

Za reševanje problema potujočega obiskovalca smo primerjali dve metodi. Prva metoda je gore omenjeni Naivni algoritem, prikazan v Algoritmu 10. Druga metoda je algoritem Koper, prikazan v Algoritmu 11. Za reševanje TSP, ki je eden izmed korakov pri obema algoritmoma, smo uporabili algoritem Concorde, ki je predstavljen v poglavju 2.2.5. Za reševanje problema najkrajše poti vseh parov smo uporabili prilagojeni Floyd-Warshallov algoritem, ki je bil predstavljen v poglavju 4.2.2.

Rezultati poskusa so predstavljeni v Tabeli 4.1. Peti stolpec Tabele 4.1, predstavlja dolžino sprehoda (cena rešitve, ki smo jo dobili pri poskusu). Stolpec se imenuje cena sprehoda (angl. *tour cost*) in v vseh šestih primerih so najkrajši sprehodi pridobljeni z algoritmom Koper. Zadnji stolpec v Tabeli 4.1, predstavlja razliko med uporabljenimi metodami, prikazano v odstotkih. Prva metoda, naivni algoritem, se je odrezal veliko slabše od algoritma Koper. Kakovost rešitev varira v intervalu od 6,52 odstotka, v primeru Belgrade163, do 354,46 odstotka, v primeru pr1002.

Namen tega raziskovanja je bil opis in rešitev novega problema v teoriji grafov, imenovanega problem potujočega obiskovalca. Čeprav je nov problem podoben dobro znanemu problemu trgovskega potnika, ko ga poskušamo rešiti z Naivnim algoritmom, so rezultati daleč od optimalnih. V vseh testiranih primerih problema potujočega obiskovalca, algoritem Koper občutno prekaša Naivni algoritem.

5.5 Metodologija

Glavno orodje za opis in definicijo problema potujočega obiskovalca je teorija grafov. Za realistično predstavitev vozlišč našega grafa, kot tudi povezav ter cen je uporabljen geografski informacijski sistem "Google Earth", čigar podatkovno bazo smo uporabili za pridobitev mestnih znamenitosti, križišč in sprehajalnih poti. Pomembno vlogo za dokazovanje hipoteze 1 ima platforma za raziskovanje genetskih algoritmov "EA Visualizer" [15], aplikacija napisana v programskem jeziku Java.

Za dokazovanje hipoteze 2 smo uporabili in ustrezno nadgradili Floyd-Warshallov algoritem [28], za iskanje najkrajših poti med vsemi pari vozlišč grafa $G = (V, E, c)$. Uporabili smo tudi znane optimizacijske metode za reševanje simetričnega in asimetričnega problema trgovskega potnika, presek-ravnine [114, 116], na čigar osnovi temeljijo metode Concorde [3–5] in hevristične metode Lin-Kernighan algoritma [6, 46, 70, 79]. Obe aplikaciji sta pisani v programskem jeziku AnsiC. Poleg tega za dokazovanje hipoteze 2 smo uporabili znanje iz matematičnih modelov znanih kot problem linearne programiranja (angl. *linear programming problems*) [23, 132], kot

tudi simpleks metode (angl. *simplex methods*) [32].

5.6 Doprinos k znanosti

Doprinos k znanosti predstavljajo naslednji rezultati:

- Izdelava cepljenega genetskega algoritma za reševanje problema trgovskega potnika.
- Potrditev, da problem trgovskega potnika lahko uspešno rešimo z uporabo cepljenega genetskega algoritma.
- Izdelava posebne metode za reševanje problema potujočega obiskovalca.
- Izdelava realnih primerov problema potujočega obiskovalca za mesta Koper, Beograd in Benetke.
- Potrditev da vsak primer problema potujočega obiskovalca, ki je rešen z posebno metodo, predstavlja zelo zadovoljivo rešitev.

Rezultati doktorske disertacije predstavljajo doprinos k premoščanju razlike med teoretičnim računalništvom in njegovo uporabo v praksi: boljšemu razumevanju in modeliranju realnih problemov iz gospodarstva, predstavljenih kot NP-težki problemi v teoriji grafov, kot tudi doprinos optimizacijskim metodam za reševanje omenjenih problemov.

Naj omenimo še, da so rezultati disertacije objavljeni v naslednjih znanstvenih člankih:

- M. Djordjevic, Influence of Grafting a Hybrid Searcher Into the Evolutionary Algorithm, *Proceedings of the 17th International Electrotechnical and Computer Science Conference, Portoroz, Slovenia* (2008), 115–118.
- M. Djordjevic, M. Tuba, and B. Djordjevic, Impact of Grafting a 2-opt Algorithm Based Local Searcher Into the Genetic Algorithm, *Proceedings of the 9th WSEAS international conference on Applied informatics and communications, AIC 2009, Moscow, Russia* (2009), 485–490.
- M. Djordjevic, and A. Brodnik, Quantitative Analysis of Separate and Combined Performance of Local Searcher and Genetic Algorithm, *Book of Abstract of International Conference on Operations Research, OR 2011, Zurich, Switzerland* (2011), 130.
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- M. Djordjevic, M. Grgurovic and A. Brodnik, The Traveling Visitor Problem and the Koper Algorithm for Solving It, *Book of Abstracts of 25th Conference of European Chapter on Combinatorial Optimization, ECCO 2012, Antalya, Turkey* (2012), 10.

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- M. Djordjevic, J. Zibert, M. Grgurovic, and A. Brodnik, Methods for Solving the Traveling Visitor Problem, *Proceedings of the 1st International Internet and Business Conference, IBC 2012, Rovinj, Croatia* (2012), 174–179.
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Declaration

I declare that this PhD Thesis does not contain any materials previously published or written by another person except where due reference is made in the text.

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